

2D Oculomotor Plant Mathematical Model Equations

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Abstract: this report develops force equations for four extraocular muscles – medial, lateral recti and superior, inferior recti. Derived forces allow moving an eye globe in two dimensional plane.

1. The Oculomotor Plant mechanical model

The Oculomotor plant is represented by a mechanical model composed of six muscles attached to a globe of ice representing the eye sphere. This paper works only with the oculomotor plant that consists of the lateral (the muscle that is closer to the ear), the medial (the muscle that is closer to the nose), the superior (the muscle that is closer to the forehead) and inferior (the muscle that is closer to the cheek) recti, eye globe and surrounding tissues.

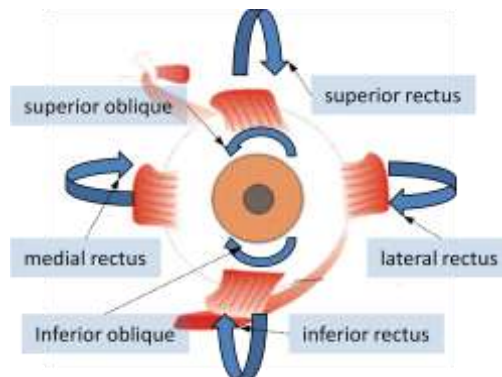


Figure 1. Left eye diagram front view

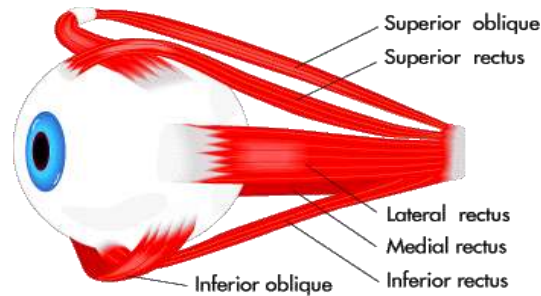


Figure 2. Left eye diagram side view

The eye globe's radius is 11mm. The lateral, medial, superior and the inferior recti are modeled through a system of mechanical components described in latter sub-sections. Each muscle can play the role of the agonist or the antagonist. The agonist muscle pulls the eye globe in the required direction and the antagonist muscle resists the pull. Each of the following section is modeled based on the role that each muscle plays in a particular situation. As an example when the eye globe moves to the direction of upper right direction, the lateral/superior recti support for the movement and medial/inferior recti resist the movement based on the role of the muscle at that instance. Lateral/superior plays the agonist and medial/inferior plays the antagonist role in this particular situation. Evoked by muscle movement, an eye can move in eight different directions : Right horizontal, Left horizontal, Top vertical, Bottom vertical, Right upward, Left upward, Right downward and Left downward. This paper discusses only Right upward movement and Left Downward as examples, but the model can be modified to allow eye globe movements in all directions.

2. Two Dimensional Oculomotor Plant Mathematical Model (2DOPMM)

The OPMM is consist of four contour points, each with eye muscle attached to eye globe which provides forces to rotate eye globe around the socket through the use of four muscles, lateral rectus, superior rectus, medial rectus and inferior rectus. The subscript notation will identify with LR the parameters that belong to the lateral rectus, with MR the parameters that

belong to the medial rectus, with SR the parameters that belong to the superior rectus and IR the parameters that belong to the inferior rectus. Parameters without those subscripts are identical to both types of muscles. The figure illustrates the eye in the coordination position (0, 0) with respect to the eye center axis in the rest position, with muscles compensate the reactions of each muscle and stabilize the eye in the coordination framework.

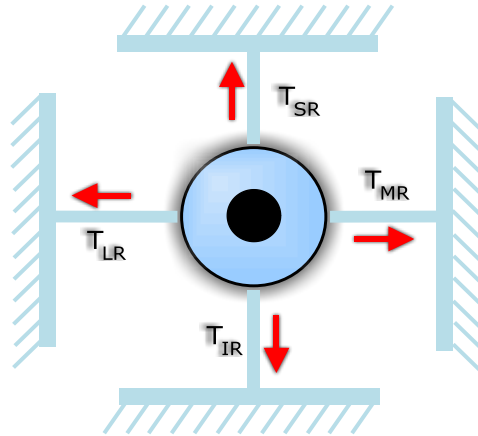


Figure 3. Oculomotor Plant Mathematical Model with four muscle forces

When eye moves to a particular position from the coordination position (0, 0) as one of eight basic movement types listed above, each muscle connected to the eye globe contract or stretch accordingly.

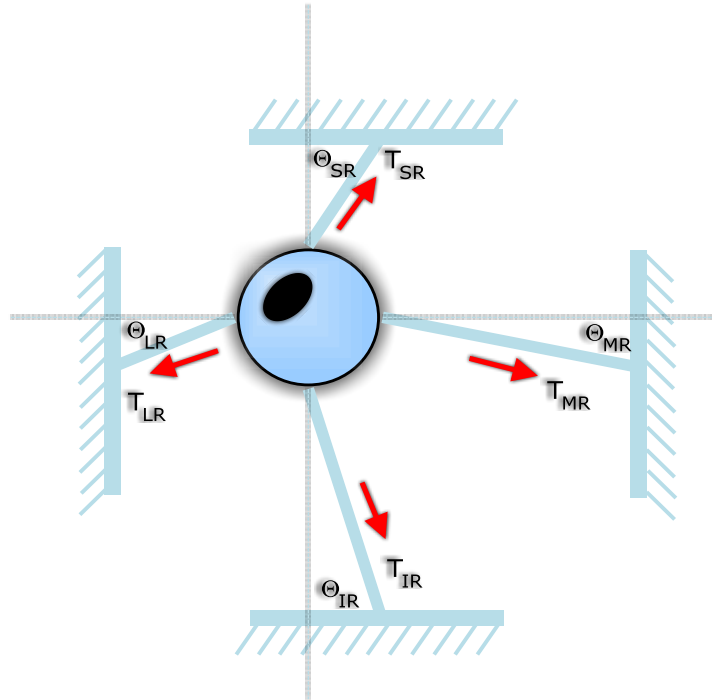


Figure 4. Positive Upward movement with muscle forces

Consider a positive upward movement of the right eye, which move the eye's visual axis from the coordination position $(0, 0)$. The lateral rectus and the superior rectus provide forces to move the muscle to the required position and stable on the target visual axis. The muscle medial rectus and inferior rectus stretches and contest the movement in the required direction. When eye fixate upon the target visual axis, muscles compensate the forces and stabilize on the new visual axis. Rotations of each muscle because of the horizontal and vertical movement of eye from its origin, Θ_{HR} and Θ_{VR} makes a rotation angle in with respect to each of muscle connected to the eye globe, by lateral rectus, superior rectus, medial rectus and inferior rectus as Θ_{LR} , Θ_{SR} , Θ_{MR} , Θ_{IR} respectively.

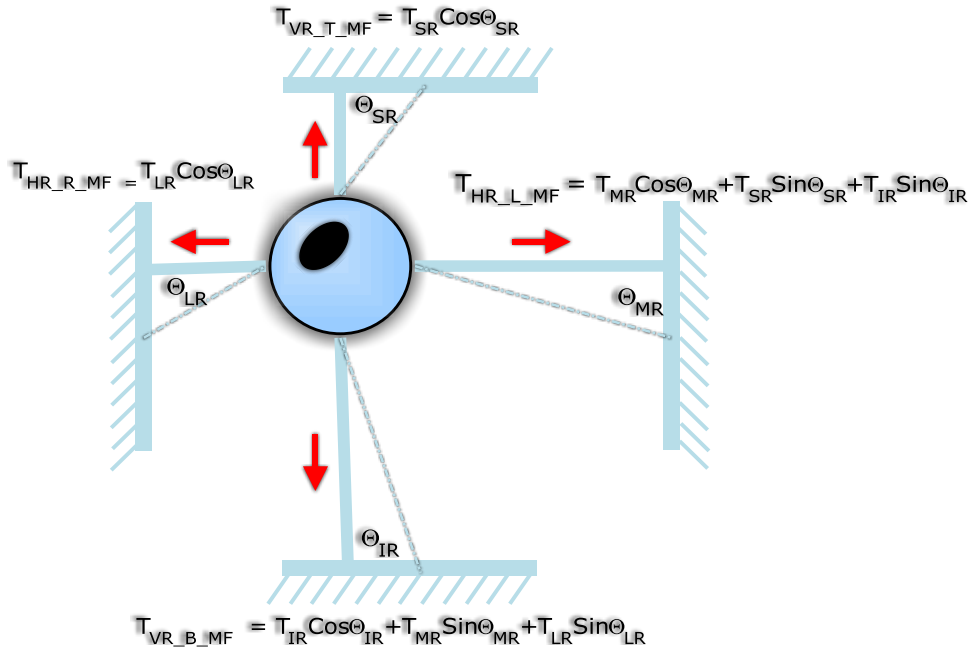






Figure 5. Positive Upward movement with vertically and horizontally projected muscle forces

Each muscle provides projections of its forces activated by the neuronal control signal, at each point in the eye globe in basic eye movement types. The projections of each muscle force is directed according to the direction of its basic movement and this provides four basic muscle force equations at each point of connection at the eye globe as,

Horizontal Right Muscle Force ($T_{HR_R_MF}$)	: $T_{LR} \cos \theta_{LR}$	
Horizontal Left Muscle Force ($T_{HR_L_MF}$)	: $T_{SR} \cos \theta_{SR}$	
Vertical Top Muscle Force ($T_{VR_T_MF}$)	: $T_{MR} \cos \theta_{MR} + T_{SR} \sin \theta_{SR} + T_{IR} \sin \theta_{IR}$	
Vertical Bottom Muscle Force ($T_{VR_B_MF}$)	: $T_{IR} \cos \theta_{IR} + T_{MR} \sin \theta_{MR} + T_{LR} \sin \theta_{LR}$	

3. Positive Upward Eye Movement

The lateral rectus MMM, holding the eye in a fixation state, is represented by Figure 6. Neuronal control signal N_{LR} creates active tension force \vec{F}_{HR_LR} that works in parallel with the

length-tension force $\vec{F}_{HR_LT_LR}$. Altogether they produce tension $\vec{T}_{HR_R_MF} = \vec{F}_{HR_LR} + \vec{F}_{HR_LT_LR}$ that is propagated through the series elasticity components to the eye globe $\vec{T}_{HR_R_MF} = \vec{F}_{HR_SE_LR}$.

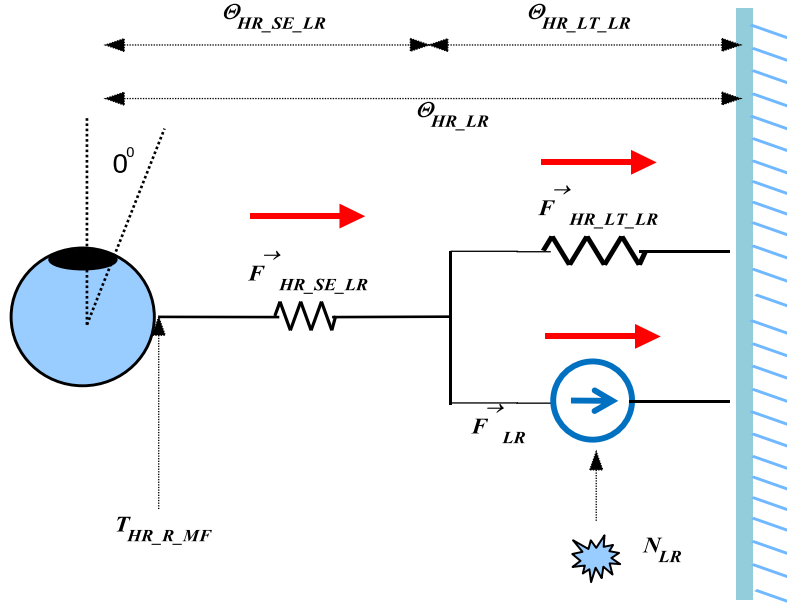


Figure 6. Muscle Mechanical Model. Red arrows represent direction of the forces

Scalar values of the forces are as following: Length tension force of lateral rectus is $F_{HR_LT_LR} = K_{LT}\theta_{HR_LT_LR}$ where $\theta_{HR_SE_LR}$ is the displacement of the spring in the horizontal direction and K_{SE} is the spring's coefficient, and force propagated by the series elasticity component is $T_{LR}=K_{SE}\theta_{HR_SE_LR}$, where $\theta_{HR_SE_LR}$ is the displacement of the spring in the horizontal direction and K_{SE} is the spring's coefficient.

Tension \vec{T}_{LR} applied by lateral rectus to the eye globe is counterbalanced by tension of medial rectus, superior rectus and inferior rectus $\vec{T}_{MR} + \vec{T}_{SR} + \vec{T}_{IR}$ and tension created by passive elasticity of those muscles and the tissue surrounding the eye globe $\overrightarrow{K_p\Delta\theta}$. The MMM becomes

more complex during eye rotations, making it to present MMMs for the horizontal and vertical muscle forces. To provide more detail, muscle forces are presented with scalar values.

3.1. Horizontal Right Muscle Force (HR_R_MF)

The agonist muscle contracts, rotates the eye globe and stretches the antagonist muscle. Assuming that the lateral rectus play the role of the agonist, Figure 7 presents the MMM of the Horizontal Right Muscle Force (HR_R_MF), pulling the eye-globe in the positive direction.

Lets assume that prior to the eye movement, the length of the displacement in the series elasticity and the length tension spring components in the horizontal direction added together is θ_{HR_LR} . Considering that the right eye moves to the right by $\Delta\theta_{HR}$ degrees, the original displacement θ_{HR_LR} is reduced making the resulting displacement $\theta_{HR_LR} - \Delta\theta_{HR}$. The displacement $\Delta\theta_{HR}$ can be broken in the displacement inside of is $\Delta\theta_{HR} = \Delta\theta_{HR_SE_LR} - \Delta\theta_{HR_LT_LR}$. Muscle contraction expands the series elastic component making the resulting displacement $\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}$. Muscle contraction shortens the length tension component making the resulting displacement $\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}$. The damping component modeling the force velocity relationship $B_{AG}\Delta\dot{\theta}_{LT_LR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based upon the velocity of contraction of the length tension component.

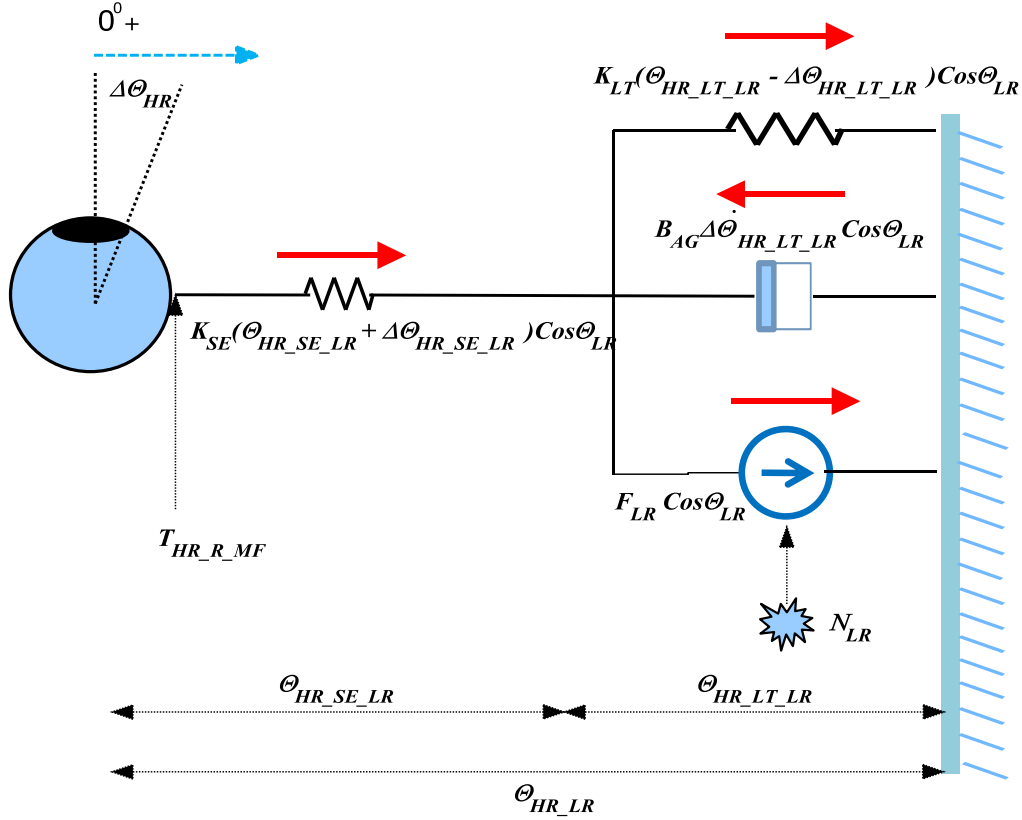


Figure 7. Horizontal Right Muscle Force mechanical model. Arrows represent direction of the forces.

Using Figure 7, we can write the equation of force with which the part of the diagram responsible for contraction by the lateral rectus (active state tension, damping component, length tension component) pulls the series elasticity component.

$$T_{HR_R_MF} = F_{LR} \cos \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) \cos \theta_{LR} - B_{AG}\dot{\theta}_{HR_LT_LR} \cos \theta_{LR} \quad (1)$$

Resisting the contraction, the series elasticity components of lateral rectus propagates the contractile force by pulling the eye globe with the same force $T_{HR_R_MF}$.

$$T_{HR_R_MF} = K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \cos \theta_{LR} \quad (2)$$

Equations **Error! Reference source not found.** and **Error! Reference source not found.** can be used to calculate the force $T_{HR_R_MF}$ in terms of the eye rotation $\Delta\theta_{HR}$ and displacement $\Delta\theta_{HR_LT_LR}$ of the length tension component of the muscle.

$$-K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \cos \theta_{LR} + F_{LR} \cos \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) \cos \theta_{LR} - B_{AG} \Delta\dot{\theta}_{HR_LT_LR} \cos \theta_{LR} = 0$$

Taking into consideration that $\theta_{HR_LR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_LT_LR} - \Delta\theta_{HR_SE_LR}$

the following equation can be calculated:

$$\theta_{HR_LR} - \Delta\theta_{HR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_SE_LR}$$

$$\theta_{HR_LR} - \Delta\theta_{HR} - \theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR} = \theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}$$

$$-K_{SE}(\theta_{HR_LR} - \Delta\theta_{HR} - \theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) + F_{LR} + K_{LT}(\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) - B_{AG} \Delta\dot{\theta}_{HR_LT_LR} = 0$$

$$-K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) + (F_{LR} - K_{SE}(\theta_{HR_LR} - \theta_{HR_LT_LR}) + K_{LT}\theta_{HR_LT_LR}) - K_{LT}\Delta\theta_{HR_LT_LR} - B_{AG}\Delta\dot{\theta}_{HR_LT_LR} = 0$$

$$\text{Assigning } \hat{F}_{LR} = F_{LR} - K_{SE}(\theta_{HR_LR} - \theta_{HR_LT_LR}) + K_{LT}\theta_{HR_LT_LR}$$

$$K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) = \hat{F}_{LR} - K_{LT}\Delta\theta_{HR_LT_LR} - B_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

New equation for $T_{HR_R_MF}$ can be written as:

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})$$

$$T_{HR_R_MF} = \hat{F}_{LR} - K_{LT}\Delta\theta_{HR_LT_LR} - B_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

$$\frac{T_{HR_R_MF}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR_LT_LR}$$

$$T_{HR_R_MF} = \hat{F}_{LR} - K_{LT} \left(\frac{T_{HR_R_MF}}{K_{SE}} + \Delta\theta_{HR} \right) - B_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{HR_R_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_LR} \quad (3)$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) \quad (4)$$

3.2. Horizontal Left Muscle Force (HR_L_MF)

The antagonist muscle is stretched by the agonist pull. Assuming that the projections of medial rectus, and inferior rectus play the role of the antagonist, and superior rectus plays the agonist, MMM of the Horizontal Left Muscle Force in the positive direction can be represented in Figure 8

By considering the medial rectus, originally the length of the displacement in the series elasticity and the length tension springs added together is θ_{HR_MR} . θ_{HR_MR} increases when the eye moves to the right by $\Delta\theta_{HR}$, making the resulting displacement $\theta_{HR_MR} + \Delta\theta_{HR}$. Both length tension and series elasticity components lengthen as a result of the agonist pull. The eye rotation $\Delta\theta_{HR}$ can be split into the displacement of the series elasticity component and the length tension component: $\Delta\theta_{HR} = \Delta\theta_{HR_SE_MR} + \Delta\theta_{HR_LT_MR}$. The resulting displacement for the series elasticity component is $\theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}$ and for the length tension component is $\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}$. The damping component modeling the force velocity relationship $B_{ANT}\Delta\dot{\theta}_{LT_MR}$ resists the muscle stretching. The amount of resistive force is based upon the velocity of stretching of the length tension component.

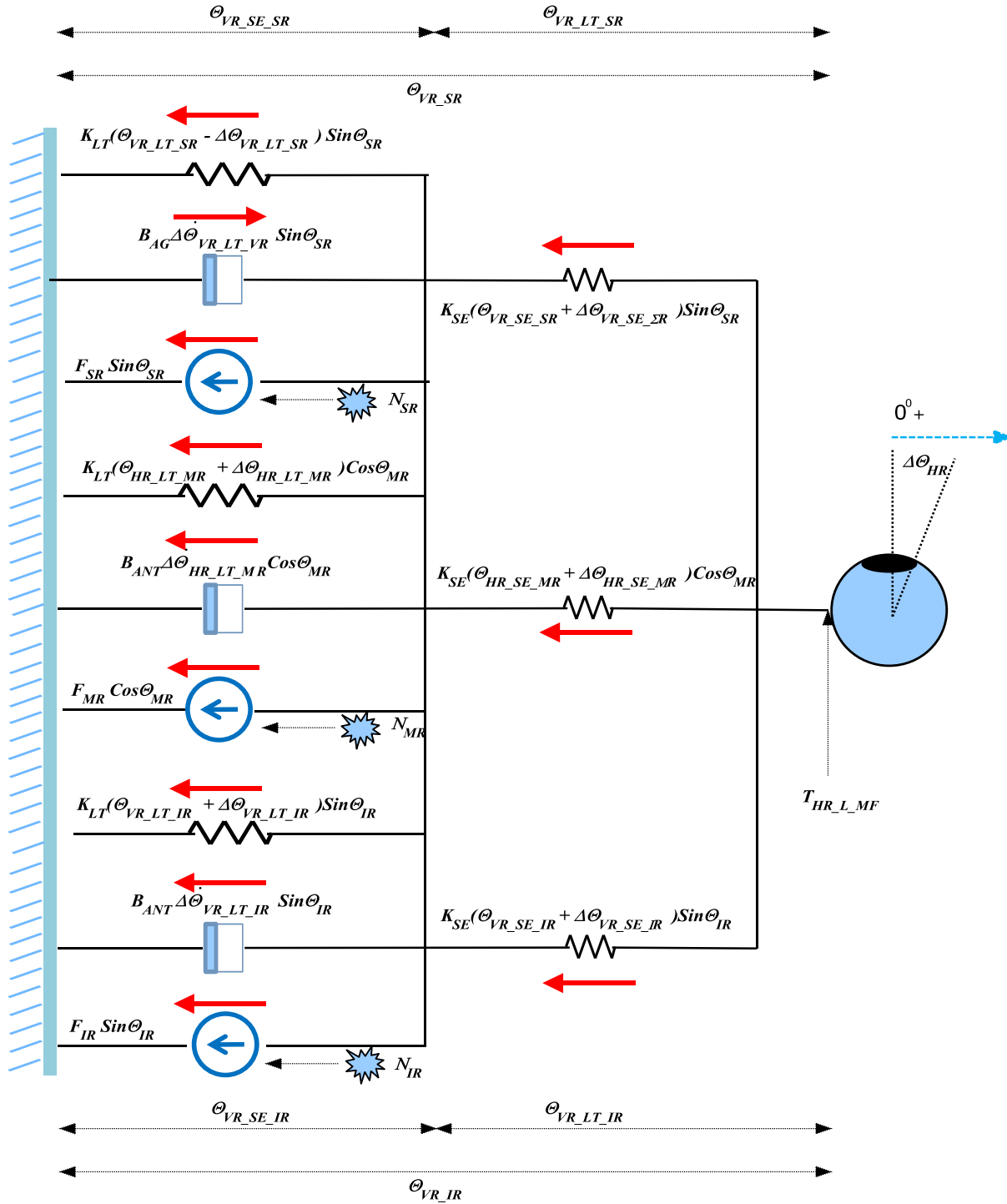


Figure 8. Horizontal Left Muscle Force. Arrows represent the direction of the forces.

Considering that the right eye moves upward $\Delta\theta_{VR}$ degrees, the original displacement θ_{VR_SR} is reduced making the resulting displacement $\theta_{VR_SR} - \Delta\theta_{SR}$. The displacement $\Delta\theta_{SR}$ can be broken in the displacement inside of is $\Delta\theta_{SR} = \Delta\theta_{VR_SE_SR} - \Delta\theta_{VR_LT_SR}$. Muscle contraction expands the series elastic component making the resulting displacement $\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}$. Muscle contraction shortens the length tension component making the resulting displacement $\theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR}$. The damping component modeling the force velocity relationship $B_{AG}\Delta\dot{\theta}_{LT_SR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based upon the velocity of contraction of the length tension component.

By considering the inferior rectus, originally the length of the displacement in the series elasticity and the length tension springs added together is θ_{VR_IR} . θ_{VR_IR} increases when the eye moves to the up by $\Delta\theta_{VR}$, making the resulting displacement $\theta_{VR_IR} + \Delta\theta_{VR}$. Both length tension and series elasticity components lengthen as a result of the agonist pull. The eye rotation $\Delta\theta_{VR}$ can be split into the displacement of the series elasticity component and the length tension component: $\Delta\theta_{VR} = \Delta\theta_{VR_SE_IR} + \Delta\theta_{VR_LT_IR}$. The resulting displacement for the series elasticity component is $\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}$ and for the length tension component is $\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}$. The damping component modeling the force velocity relationship $B_{ANT}\Delta\dot{\theta}_{LT_IR}$ resists the muscle stretching. The amount of resistive force is based upon the velocity of stretching of the length tension component.

We can write equation of forces with which the part of the diagram responsible for the contraction (active state tensions, damping components, length tension components) pulls the series elasticity components.

$$T_{MR} = -F_{MR} \cos \theta_{MR} - K_{LT}(\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}) \cos \theta_{MR} - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR} \cos \theta_{MR}$$

$$T_{IR} = -F_{IR} \sin \theta_{IR} - K_{LT}(\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}) \sin \theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_IR} \sin \theta_{IR}$$

$$T_{SR} = -F_{SR} \sin \theta_{SR} - K_{LT}(\theta_{VR_{LT}_{SR}} - \Delta\theta_{VR_{LT}_{SR}}) \sin \theta_{SR} + B_{AG} \Delta\dot{\theta}_{VR_{LT}_{SR}} \sin \theta_{SR}$$

$$T_{MR} = -K_{SE}(\theta_{HR_{SE}_{MR}} + \Delta\theta_{HR_{SE}_{MR}}) \cos \theta_{MR}$$

$$T_{IR} = -K_{SE}(\theta_{VR_{SE}_{IR}} + \Delta\theta_{VR_{SE}_{IR}}) \sin \theta_{IR}$$

$$T_{SR} = -K_{SE}(\theta_{VR_{SE}_{SR}} + \Delta\theta_{VR_{SE}_{SR}}) \sin \theta_{SR}$$

Above six equations can be used to calculate forces T_{MR} , T_{IR} , T_{SR} in terms of the eye rotation $\Delta\theta_{HR}$, $\Delta\theta_{VR}$ and displacements $\Delta\theta_{HR_{LT}_{MR}}$, $\Delta\theta_{VR_{LT}_{IR}}$, $\Delta\theta_{VR_{LT}_{SR}}$ of the length tension components of each muscle respectively.

3.2.1. Medial Recuts Muscle Force (T_{MR}) of the Horizontal Left Muscle Force

$$\begin{aligned} K_{SE}(\theta_{HR_{SE}_{MR}} + \Delta\theta_{HR_{SE}_{MR}}) \cos \theta_{MR} - F_{MR} \cos \theta_{MR} - K_{LT}(\theta_{HR_{LT}_{MR}} + \Delta\theta_{HR_{LT}_{MR}}) \cos \theta_{MR} \\ - B_{ANT} \Delta\dot{\theta}_{HR_{LT}_{MR}} \cos \theta_{MR} = 0 \end{aligned}$$

Taking into consideration that $\theta_{HR_{MR}} = \theta_{HR_{LT}_{MR}} + \theta_{HR_{SE}_{MR}}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_{LT}_{MR}} + \Delta\theta_{HR_{SE}_{MR}}$

the following equation can be calculated:

$$\theta_{HR_{MR}} + \Delta\theta_{HR} = \theta_{HR_{LT}_{MR}} + \theta_{HR_{SE}_{MR}} + \Delta\theta_{HR_{LT}_{MR}} + \Delta\theta_{HR_{SE}_{MR}}$$

$$\theta_{HR_{MR}} + \Delta\theta_{HR} - \theta_{HR_{LT}_{MR}} - \Delta\theta_{HR_{LT}_{MR}} = \theta_{HR_{SE}_{MR}} + \Delta\theta_{HR_{SE}_{MR}}$$

$$K_{SE}(\theta_{HR_{MR}} + \Delta\theta_{HR} - \theta_{HR_{LT}_{MR}} - \Delta\theta_{HR_{LT}_{MR}}) - F_{MR} - K_{LT}(\theta_{HR_{LT}_{MR}} + \Delta\theta_{HR_{LT}_{MR}}) - B_{ANT} \Delta\dot{\theta}_{HR_{LT}_{MR}} = 0$$

$$\begin{aligned} K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_{LT}_{MR}}) - F_{MR} - K_{SE}\theta_{HR_{LT}_{MR}} + K_{SE}\theta_{HR_{MR}} - K_{LT}\theta_{HR_{LT}_{MR}} - K_{LT}\Delta\theta_{HR_{LT}_{MR}} \\ - B_{ANT} \Delta\dot{\theta}_{HR_{LT}_{MR}} = 0 \end{aligned}$$

$$\text{Assigning } \hat{F}_{MR} = F_{MR} + K_{SE}\theta_{HR_{LT}_{MR}} - K_{SE}\theta_{HR_{MR}} + K_{LT}\theta_{HR_{LT}_{MR}}$$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_{LT}_{MR}}) = \hat{F}_{MR} + K_{LT}\Delta\theta_{HR_{LT}_{MR}} + B_{ANT} \Delta\dot{\theta}_{HR_{LT}_{MR}}$$

$$-K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR_LT_MR} - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

New equation for T_{MR} can be written as:

$$T_{MR} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR})$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR_LT_MR} - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR_LT_MR}$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\left(\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR}\right) - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{MR} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{MR} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR})$$

3.2.2. Inferior Recuts Muscle Force (T_{IR}) of the Horiztonal Left Muscle Force

$$K_{SE}(\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}) \sin \theta_{IR} - F_{IR} \sin \theta_{IR} - K_{LT}(\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}) \sin \theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_IR} \sin \theta_{IR} = 0$$

Taking into consideration that $\theta_{VR_IR} = \theta_{VR_LT_IR} + \theta_{VR_SE_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_SE_IR}$

the follwing equation can be calculated:

$$\theta_{VR_IR} + \Delta\theta_{VR} = \theta_{VR_LT_IR} + \theta_{VR_SE_IR} + \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_SE_IR}$$

$$\theta_{VR_IR} + \Delta\theta_{VR} - \theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR} = \theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}$$

$$K_{SE}(\theta_{VR_IR} + \Delta\theta_{VR} - \theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) - F_{IR} - K_{LT}(\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}) - B_{ANT}\Delta\dot{\theta}_{VR_LT_IR} = 0$$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR}) - F_{IR} - K_{SE}\theta_{VR_LT_JR} + K_{SE}\theta_{VR_JR} - K_{LT}\theta_{VR_LT_JR} - K_{LT}\Delta\theta_{VR_LT_JR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR} = 0$$

Assigning $\hat{F}_{IR} = F_{IR} + K_{SE}\theta_{VR_LT_JR} - K_{SE}\theta_{VR_JR} + K_{LT}\theta_{VR_LT_JR}$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR}) = \hat{F}_{IR} + K_{LT}\Delta\theta_{VR_LT_JR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$-K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR}) = -\hat{F}_{IR} - K_{LT}\Delta\theta_{VR_LT_JR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

New equation for T_{IR} can be written as:

$$T_{IR} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR})$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\Delta\theta_{VR_LT_JR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR} = \Delta\theta_{VR_LT_JR}$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\left(\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR}\right) - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$T_{IR} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$T_{IR} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR})$$

3.2.3. Superior Rectus Muscle Force (T_{SR}) of the Horizontal Left Muscle Force

$$-K_{SE}(\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}) \sin \theta_{SR} + F_{SR} \sin \theta_{SR} + K_{LT}(\theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR}) \sin \theta_{SR} - B_{AG}\Delta\dot{\theta}_{VR_LT_SR} \sin \theta_{SR} = 0$$

Taking into consideration that $\theta_{VR_SR} = \theta_{VR_LT_SR} + \theta_{VR_SE_SR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR_LT_SR} - \Delta\theta_{VR_SE_SR}$

the following equation can be calculated:

$$\theta_{VR_SR} - \Delta\theta_{VR} = \theta_{VR_LT_SR} + \theta_{VR_SE_SR} - \Delta\theta_{VR_LT_SR} + \Delta\theta_{VR_SE_SR}$$

$$\theta_{VR_SR} - \Delta\theta_{VR} - \theta_{VR_LT_SR} + \Delta\theta_{VR_LT_SR} = \theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}$$

$$-K_{SE}(\theta_{VR_SR} - \Delta\theta_{VR} - \theta_{VR_LT_SR} + \Delta\theta_{VR_LT_SR}) + F_{SR} + K_{LT}(\theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR}) - B_{AG}\Delta\dot{\theta}_{VR_LT_SR} = 0$$

$$\begin{aligned} -K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR}) + (F_{SR} - K_{SE}(\theta_{VR_SR} - \theta_{VR_LT_SR}) + K_{LT}\theta_{VR_LT_SR}) - K_{LT}\Delta\theta_{VR_LT_SR} - B_{AG}\Delta\dot{\theta}_{VR_LT_SR} \\ = 0 \end{aligned}$$

Assigning $\hat{F}_{SR} = F_{SR} - K_{SE}(\theta_{VR_SR} - \theta_{VR_LT_SR}) + K_{LT}\theta_{VR_LT_SR}$

$$K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR}) = \hat{F}_{SR} - K_{LT}\Delta\theta_{VR_LT_SR} - B_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

$$-K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR}) = -\hat{F}_{SR} + K_{LT}\Delta\theta_{VR_LT_SR} + B_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

New equation for T_{SR} can be written as:

$$T_{SR} = -K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR})$$

$$T_{SR} = -\hat{F}_{SR} + K_{LT}\Delta\theta_{VR_LT_SR} + B_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

$$\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}} = \Delta\theta_{VR_LT_SR}$$

$$T_{SR} = -\hat{F}_{SR} + K_{LT}\left(\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}\right) + B_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{SR} = -\frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{SR} = -K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR})$$

3.2.4. Formulating Horizontal Left Muscle Force ($T_{HR_L_MF}$)

$$T_{HR_L_MF} = T_{MR} + T_{IR} + T_{SR}$$

$$T_{HR_L_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_MR} - \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_IR} - \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{HR_L_MF} = -\frac{(\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_MR} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) - K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR}) - K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR})$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) + K_{SE}\Delta\theta_{VR_LT_IR} - K_{SE}\Delta\theta_{VR_LT_SR}$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_SR})$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_SR}) \quad (5)$$

$$T_{HR_L_MF} = -\frac{(\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_MR} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR} \quad (6)$$

3.3. Vertical Top Muscle Force (VR_T_MF)

Considering the vertical movement of the eye in the right upward eye movement, assuming that the superior rectus play the role of the agonist, MMM of the Vertical Top Muscle Force (HR_T_MF), pulling the eye-globe in the positive vertical direction represented in the Figure 9.

Lets assume that prior to the eye movement, the length of the displacement in the series elasticity and the length tension spring components in the vertical direction added together is θ_{VR_SR} . Considering that the right eye moves to the upward by $\Delta\theta_{VR}$ degrees, the original displacement θ_{VR_SR} is reduced making the resulting displacement $\theta_{VR_SR} - \Delta\theta_{VR}$. The displacement $\Delta\theta_{VR}$ can be broken in the displacement inside of is $\Delta\theta_{VR} = \Delta\theta_{VR_SE_SR} - \Delta\theta_{VR_LT_SR}$. Muscle contraction expands the series elastic component making the resulting displacement $\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}$. Muscle contraction shortens the length tension component making the resulting displacement $\theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR}$. The damping component modeling the force velocity relationship $B_{AG}\Delta\dot{\theta}_{LT_SR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based upon the velocity of contraction of the length tension component.

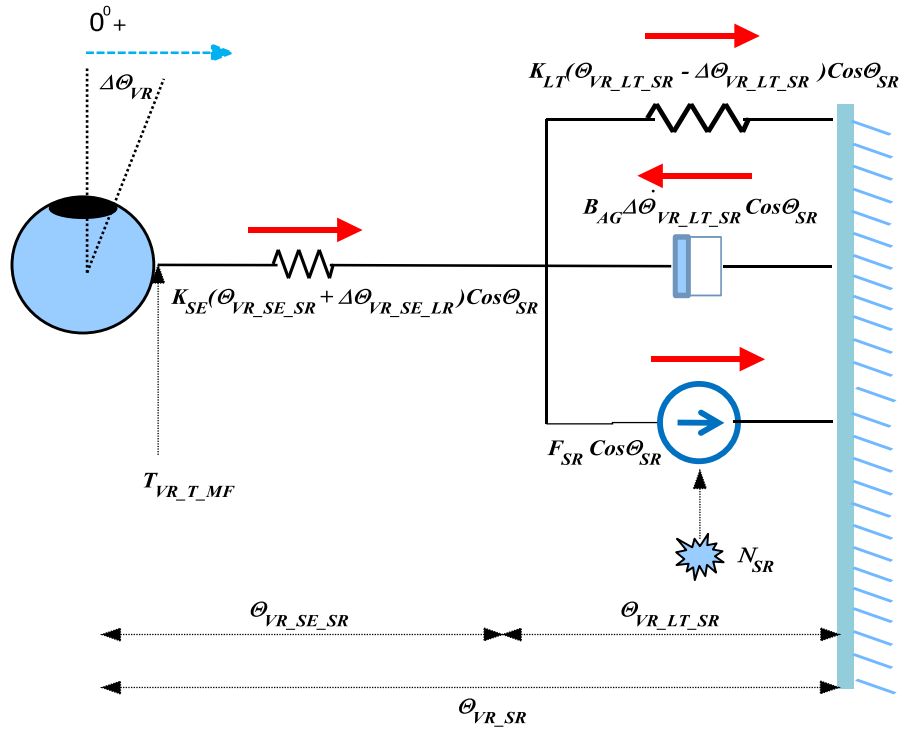


Figure 9. Vertical Top Muscle Force. Arrows represent the direction of forces.

We can write the equation of force with which the part of the diagram responsible for contraction by the superior rectus (active state tension, damping component, length tension component) pulls the series elasticity component.

$$T_{VR_T_SR} = F_{SR} \cos \theta_{SR} + K_{LT}(\theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR}) \cos \theta_{SR} - B_{AG}\Delta\dot{\theta}_{LT_SR} \cos \theta_{SR} \quad (7)$$

Resisting the contraction, the series elasticity components of superior rectus propagates the contractile force by pulling the eye globe with the same force $T_{VR_T_MF}$.

$$T_{VR_T_MF} = K_{SE}(\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}) \cos \theta_{SR} \quad (8)$$

Equations (7) and (8) can be used to calculate the force $T_{VR_T_MF}$ in terms of the eye rotation $\Delta\theta_{VR}$ and displacement $\Delta\theta_{VR_LT_SR}$ of the length tension component of the muscle.

$$T_{VR_T_MF} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{LT_SR} \quad (9)$$

$$T_{VR_T_MF} = K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR}) \quad (10)$$

3.4. Vertical Bottom Muscle Force (VR_B_MF)

Assuming that the projections of medial rectus, and inferior rectus play the role of the antagonist, and lateral rectus plays the agonist, the MMM of the Vertical Bottom Muscle Force in the positive direction represented in the Figure 10.

By considering the medial rectus, originally the length of the displacement in the series elasticity and the length tension springs added together is θ_{HR_MR} . θ_{HR_MR} increases when the eye moves to the right by $\Delta\theta_{VR}$, making the resulting displacement $\theta_{HR_MR} + \Delta\theta_{HR}$. Both length

tension and series elasticity components lengthen as a result of the agonist pull. The eye rotation $\Delta\theta_{VR}$ can be split into the displacement of the series elasticity component and the length tension component: $\Delta\theta_{HR} = \Delta\theta_{HR_SE_MR} + \Delta\theta_{HR_LT_MR}$. The resulting displacement for the series elasticity component is $\theta_{HR_SE_VR} + \Delta\theta_{HR_SE_MR}$ and for the length tension component is $\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}$. The damping component modeling the force velocity relationship $B_{ANT}\Delta\dot{\theta}_{LT_MR}$ resists the muscle stretching. The amount of resistive force is based upon the velocity of stretching of the length tension component.

Considering that the right eye moves to the right by $\Delta\theta_{HR}$ degrees, the original displacement lateral rectus, θ_{HR_LR} is reduced making the resulting displacement $\theta_{HR_LR} - \Delta\theta_{LR}$. The displacement $\Delta\theta_{LR}$ can be broken in the displacement inside of is $\Delta\theta_{SR} = \Delta\theta_{VR_SE_SR} - \Delta\theta_{VR_LT_SR}$. Muscle contraction expands the series elastic component making the resulting displacement $\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}$. Muscle contraction shortens the length tension component making the resulting displacement $\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}$. The damping component modeling the force velocity relationship $B_{AG}\Delta\dot{\theta}_{LT_LR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based upon the velocity of contraction of the length tension component.

By considering the inferior rectus, originally the length of the displacement in the series elasticity and the length tension springs added together is θ_{VR_IR} . θ_{VR_IR} increases when the eye moves to the up by $\Delta\theta_{VR}$, making the resulting displacement $\theta_{VR_IR} + \Delta\theta_{VR}$. Both length tension and series elasticity components lengthen as a result of the agonist pull. The eye rotation $\Delta\theta_{VR}$ can be split into the displacement of the series elasticity component and the length tension component: $\Delta\theta_{VR} = \Delta\theta_{VR_SE_IR} + \Delta\theta_{VR_LT_IR}$. The resulting displacement for the series elasticity component is $\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}$ and for the length tension component is $\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}$.

The damping component modeling the force velocity relationship $B_{ANT}\Delta\dot{\theta}_{LT_IR}$ resists the muscle stretching. The amount of resistive force is based upon the velocity of stretching of the length tension component.

We can write the equation of force with which the part of the diagram responsible for the contraction (active state tensions, damping components, length tension components) pulls the series elasticity components.

$$T_{MR} = -F_{MR} \sin \theta_{MR} - K_{LT}(\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}) \sin \theta_{MR} - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR} \sin \theta_{MR}$$

$$T_{IR} = -F_{IR} \cos \theta_{IR} - K_{LT}(\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}) \cos \theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_IR} \cos \theta_{IR}$$

$$T_{LR} = -F_{LR} \sin \theta_{LR} - K_{LT}(\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) \sin \theta_{LR} + B_{AG}\Delta\dot{\theta}_{HR_LT_LR} \sin \theta_{LR}$$

$$T_{MR} = -K_{SE}(\theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}) \sin \theta_{MR}$$

$$T_{IR} = -K_{SE}(\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}) \cos \theta_{IR}$$

$$T_{LR} = -K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \sin \theta_{LR}$$

Above six equations can be used to calculate forces T_{MR} , T_{IR} , T_{LR} in terms of the eye rotation $\Delta\theta_{HR}$, $\Delta\theta_{VR}$ and displacements $\Delta\theta_{HR_LT_MR}$, $\Delta\theta_{VR_LT_IR}$, $\Delta\theta_{HR_LT_LR}$ of the length tension components of each muscle respectively.

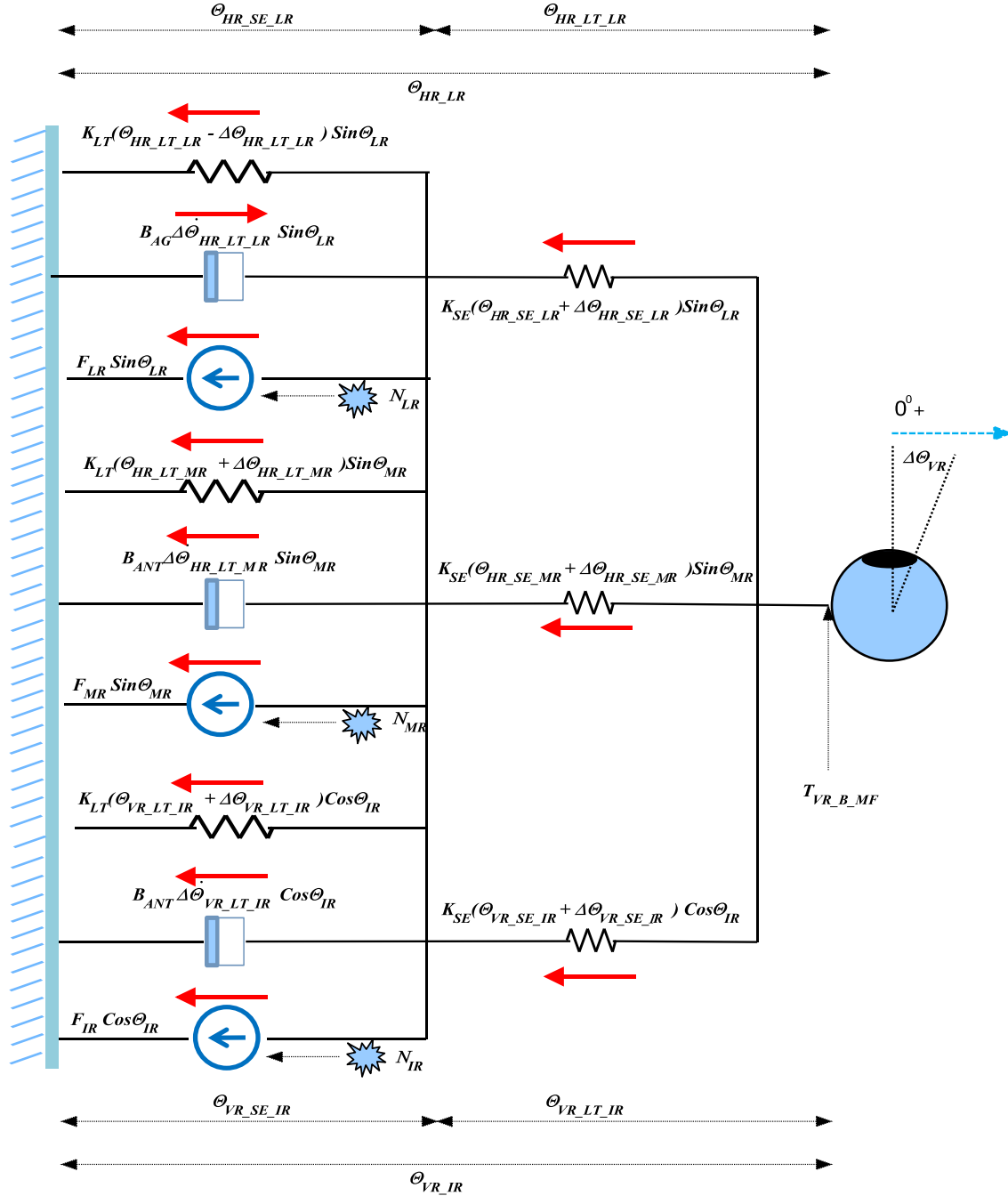


Figure 10. Vertical Top Muscle Force. Arrows represent the direction of the forces.

3.4.1. Medial Recus Muscle Force (T_{MR}) of the Vertical Bottom Muscle Force

$$\begin{aligned}
 &K_{SE}(\theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}) \sin \theta_{MR} - F_{MR} \sin \theta_{MR} - K_{LT}(\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}) \sin \theta_{MR} \\
 &\quad - B_{ANT} \Delta\dot{\theta}_{HR_LT_MR} \sin \theta_{MR} = 0
 \end{aligned}$$

Taking into consideration that $\theta_{HR_MR} = \theta_{HR_LT_MR} + \theta_{HR_SE_MR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_SE_MR}$

the following equation can be calculated:

$$\theta_{HR_MR} + \Delta\theta_{HR} = \theta_{HR_LT_MR} + \theta_{HR_SE_MR} + \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_SE_MR}$$

$$\theta_{HR_MR} + \Delta\theta_{HR} - \theta_{HR_LT_MR} - \Delta\theta_{HR_LT_MR} = \theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}$$

$$K_{SE}(\theta_{HR_MR} + \Delta\theta_{HR} - \theta_{HR_LT_MR} - \Delta\theta_{HR_LT_MR}) - F_{MR} - K_{LT}(\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}) - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR} = 0$$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) - F_{MR} - K_{SE}\theta_{HR_LT_MR} + K_{SE}\theta_{HR_MR} - K_{LT}\theta_{HR_LT_MR} - K_{LT}\Delta\theta_{HR_LT_MR} \\ - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR} = 0$$

Assigning $\hat{F}_{MR} = F_{MR} + K_{SE}\theta_{HR_LT_MR} - K_{SE}\theta_{HR_MR} + K_{LT}\theta_{HR_LT_MR}$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) = \hat{F}_{MR} + K_{LT}\Delta\theta_{HR_LT_MR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$-K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR_LT_MR} - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

New equation for T_{MR} can be written as:

$$T_{MR} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR})$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR_LT_MR} - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR_LT_MR}$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\left(\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR}\right) - B_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{MR} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{MR} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR})$$

3.4.2. Inferior Recuts Muscle Force (T_{IR}) of the Vertical Bottom Muscle Force

$$K_{SE}(\theta_{VR_SE_JR} + \Delta\theta_{VR_SE_JR}) \cos \theta_{IR} - F_{IR} \cos \theta_{IR} - K_{LT}(\theta_{VR_LT_JR} + \Delta\theta_{VR_LT_JR}) \cos \theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR} \cos \theta_{IR} = 0$$

Taking into consideration that $\theta_{VR_JR} = \theta_{VR_LT_JR} + \theta_{VR_SE_JR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR_LT_JR} + \Delta\theta_{VR_SE_JR}$

the following equation can be calculated:

$$\theta_{VR_JR} + \Delta\theta_{VR} = \theta_{VR_LT_JR} + \theta_{VR_SE_JR} + \Delta\theta_{VR_LT_JR} + \Delta\theta_{VR_SE_JR}$$

$$\theta_{VR_JR} + \Delta\theta_{VR} - \theta_{VR_LT_JR} - \Delta\theta_{VR_LT_JR} = \theta_{VR_SE_JR} + \Delta\theta_{VR_SE_JR}$$

$$K_{SE}(\theta_{VR_JR} + \Delta\theta_{VR} - \theta_{VR_LT_JR} - \Delta\theta_{VR_LT_JR}) - F_{IR} - K_{LT}(\theta_{VR_LT_JR} + \Delta\theta_{VR_LT_JR}) - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR} = 0$$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR}) - F_{IR} - K_{SE}\theta_{VR_LT_JR} + K_{SE}\theta_{VR_JR} - K_{LT}\theta_{VR_LT_JR} - K_{LT}\Delta\theta_{VR_LT_JR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR} = 0$$

$$\text{Assigning } \hat{F}_{IR} = F_{IR} + K_{SE}\theta_{VR_LT_JR} - K_{SE}\theta_{VR_JR} + K_{LT}\theta_{VR_LT_JR}$$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR}) = \hat{F}_{IR} + K_{LT}\Delta\theta_{VR_LT_JR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$-K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR}) = -\hat{F}_{IR} - K_{LT}\Delta\theta_{VR_LT_JR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

New equation for T_{IR} can be written as:

$$T_{IR} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR})$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\Delta\theta_{VR_LT_JR} - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR} = \Delta\theta_{VR_LT_JR}$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\left(\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR}\right) - B_{ANT}\Delta\dot{\theta}_{VR_LT_JR}$$

$$T_{IR} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_IR}$$

$$T_{IR} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR})$$

3.4.3. Lateral Recuts Muscle Force (T_{LR}) of the Vertical Bottom Muscle Force

$$\begin{aligned} -K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \sin \theta_{LR} + F_{LR} \sin \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) \sin \theta_{LR} \\ - B_{AG}\Delta\dot{\theta}_{HR_LT_LR} \sin \theta_{LR} = 0 \end{aligned}$$

Taking into consideration that $\theta_{HR_LR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_LT_LR} - \Delta\theta_{HR_SE_LR}$

the following equation can be calculated:

$$\theta_{HR_LR} - \Delta\theta_{HR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_SE_LR}$$

$$\theta_{HR_LR} - \Delta\theta_{HR} - \theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR} = \theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}$$

$$-K_{SE}(\theta_{HR_LR} - \Delta\theta_{HR} - \theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) + F_{LR} + K_{LT}(\theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) - B_{AG}\Delta\dot{\theta}_{HR_LT_LR} = 0$$

$$\begin{aligned} -K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) + (F_{LR} - K_{SE}(\theta_{HR_LR} - \theta_{HR_LT_LR}) + K_{LT}\theta_{HR_LT_LR}) - K_{LT}\Delta\theta_{HR_LT_LR} - B_{AG}\Delta\dot{\theta}_{HR_LT_LR} \\ = 0 \end{aligned}$$

$$\text{Assigning } \hat{F}_{LR} = F_{LR} - K_{SE}(\theta_{HR_LR} - \theta_{HR_LT_LR}) + K_{LT}\theta_{HR_LT_LR}$$

$$-K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) = -\hat{F}_{LR} + K_{LT}\Delta\theta_{HR_LT_LR} + B_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

New equation for T_{LR} can be written as:

$$T_{LR} = -K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})$$

$$T_{LR} = -\hat{F}_{LR} + K_{LT}\Delta\theta_{HR_LT_LR} + B_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

$$\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}} = \Delta\theta_{HR_LT_LR}$$

$$T_{LR} = -\hat{F}_{LR} + K_{LT} \left(\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}} \right) + B_{AG} \Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{LR} = -\frac{\hat{F}_{LR} K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{LR} = -K_{SE} (\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})$$

3.4.4. Formulating Vertical Bottom Muscle Force ($T_{VR_B_MF}$)

$$T_{VR_B_MF} = T_{MR} + T_{IR} + T_{LR}$$

$$T_{VR_B_MF} = -K_{SE} (\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR}) - K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR}) - K_{SE} (\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})$$

$$T_{VR_B_MF} = K_{SE} \Delta\theta_{HR_LT_MR} - K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR}) - K_{SE} \Delta\theta_{HR_LT_LR}$$

$$T_{VR_B_MF} = -K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR} - \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_LR}) \quad (11)$$

$$T_{VR_B_MF} = -\frac{\hat{F}_{MR} K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR} - \frac{\hat{F}_{IR} K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR} - \frac{\hat{F}_{LR} K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{VR_B_MF} = -\frac{\hat{F}_{MR} K_{SE}}{K_{SE} + K_{LT}} - \frac{\hat{F}_{IR} K_{SE}}{K_{SE} + K_{LT}} - \frac{\hat{F}_{LR} K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR} + \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{VR_B_MF} = -\left(\frac{\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{LR}}{K_{SE} + K_{LT}} \right) K_{SE} - \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR} + \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR} \quad (12)$$

3.5. Oculomotor Plant Mechanical Model Equations

The lateral rectus as the agonist applies Horizontal Right Muscle Force (HZ_R_MF) to the eye globe that can be calculated by (3) and **Error! Reference source not found.**. Those equations can be combined together:

$$K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_LR} \quad (13)$$

The medial rectus, superior rectus and inferior rectus collectively as the antagonist, applies the Horizontal Left Muscle Force (HZ_L_MF) to the eye globe that can be calculated by (5) and (6). Those equations can be combined together:

$$\begin{aligned} K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_SR}) & \quad (14) \\ &= \frac{(\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_MR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_MR} \\ & - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR} \end{aligned}$$

Applying Newton's second law, the sum of all forces acting on the eye globe horizontally, equals the acceleration of the eye globe multiplied by the inertia of the eye globe.

$$J\Delta\theta_{HR}'' = T_{HR_R_MF} - T_{HR_L_MF} - K_p\Delta\theta_{HR} - B_p\Delta\dot{\theta}_{HR}$$

J - Eye globe's inertia, $\Delta\theta$ - eye rotation, $\Delta\dot{\theta}$ velocity of the eye rotation, $\Delta\ddot{\theta}$ eye rotation acceleration. $T_{HR_R_MF}$, and $T_{HR_L_MF}$ assigning from previous equations can be transformed into:

$$J\Delta\theta_{HR}'' = K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR}) - K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_SR}) - K_p\Delta\theta_{HR} - B_p\Delta\dot{\theta}_{HR} \quad (15)$$

The superior rectus as the agonist applies Vertical Top Muscle Force (VR_T_MF) to the eye globe that can be calculated by (9) and (10). Those equations can be combined together:

$$K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR}) = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR} \quad (16)$$

The medial rectus, lateral rectus and inferior rectus collectively as the antagonist, applies the Vertical Bottom Muscle Force (VR_B_MF) to the eye globe that can be calculated by (11)**Error! Reference source not found.** and (12). Those equations can be combined together:

$$\begin{aligned}
& K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR} - \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_LR}) \\
&= \left(\frac{\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{LR}}{K_{SE} + K_{LT}} \right) K_{SE} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR} \\
&\quad - \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR}
\end{aligned} \tag{17}$$

Applying Newton's second law, the sum of all forces acting on the eye globe horizontally, equals the acceleration of the eye globe multiplied by the inertia of the eye globe.

$$J\Delta\theta_{VR}'' = T_{VR_T_MF} - T_{VR_B_MF} - K_p\Delta\theta_{VR} - B_p\Delta\dot{\theta}_{VR}$$

J - Eye globe's inertia, $\Delta\theta$ - eye rotation, $\Delta\dot{\theta}$ velocity of the eye rotation, $\Delta\ddot{\theta}$ eye rotation acceleration. $T_{HR_R_MF}$, and $T_{HR_L_MF}$ assigning from previous equations can be transformed into:

$$\begin{aligned}
J\Delta\theta_{VR}'' &= K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR}) - K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR} - \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_LR}) - K_p\Delta\theta_{VR} \\
&\quad - B_p\Delta\dot{\theta}_{VR}
\end{aligned} \tag{18}$$

Two differential equations can be added as

$$\Delta\dot{\theta}_{HR} = \Delta\dot{\theta}_{HR} \tag{19}$$

$$\Delta\dot{\theta}_{VR} = \Delta\dot{\theta}_{VR} \tag{20}$$

The dynamics of the active state tension for the agonist muscle at every time interval,

$[t_{sac_start}, t_{AG_sac_pulse_start}] [t_{AG_sac_pulse_start}, t_{AG_sac_pulse_end}] [t_{AG_sac_pulse_end}, t_{sac_end}]$ can be personated with the following equation for horizontal movement:

$$\dot{F}_{HR_AG}(t) = \frac{N_{AG} - \hat{F}_{HR_AG}(t)}{\tau_{AG_sac}} \tag{21}$$

The dynamics of the active state tension for the antagonist muscle at every time interval

$[t_{sac_start}, t_{ANT_sac_pulse_start}] [t_{ANT_sac_pulse_start}, t_{ANT_sac_pulse_end}] [t_{ANT_sac_pulse_end}, t_{sac_end}]$ can be personated with the following equation for horizontal movement:

$$\dot{F}_{HR_ANT}(t) = \frac{N_{ANT} - \hat{F}_{HR_ANT}(t)}{\tau_{ANT_sac}} \tag{22}$$

The dynamics of the active state tension for the agonist muscle at every time interval,

$[t_{sac_start}, t_{AG_sac_pulse_start}] [t_{AG_sac_pulse_start}, t_{AG_sac_pulse_end}] [t_{AG_sac_pulse_end}, t_{sac_end}]$ can be personated with the following equation for vertical movement:

$$\dot{F}_{VR_AG}(t) = \frac{N_{AG} - \hat{F}_{VR_AG}(t)}{\tau_{AG_sac}} \quad (23)$$

The dynamics of the active state tension for the antagonist muscle at every time interval

$[t_{sac_start}, t_{ANT_sac_pulse_start}] [t_{ANT_sac_pulse_start}, t_{ANT_sac_pulse_end}] [t_{ANT_sac_pulse_end}, t_{sac_end}]$ can be personated with the following equation for vertical movement:

$$\dot{F}_{VR_ANT}(t) = \frac{N_{ANT} - \hat{F}_{VR_ANT}(t)}{\tau_{ANT_sac}} \quad (24)$$

There are twelve differential equations ((13),(14),(15),(16),(17),(18),(19),(20),(21),(22),(23),(24)) with 12 variables $\Delta\theta_{HR_LT_LR}$, $\Delta\theta_{HR_LT_MR}$, $\Delta\theta_{VR_LT_SR}$, $\Delta\theta_{VR_LT_IR}$, $\Delta\theta_{HR}$, $\Delta\theta_{VR}$, \hat{F}_{MR} , \hat{F}_{LR} , \hat{F}_{SR} , \hat{F}_{IR} , $\Delta\dot{\theta}_{HR}$, $\Delta\dot{\theta}_{VR}$

3.6. Oculomotor Plant Mechanical Model Equations in Vector Format

$$\vec{T}_{HR_R_MF} = \vec{F}_{LR} + \vec{F}_{HR_R_MF} + \vec{F}_{AG_LR} \quad (25)$$

$$\vec{T}_{HR_L_MF} = \vec{F}_{MR} + \vec{F}_{IR} + \vec{F}_{SR} + \vec{F}_{HR_L_MF} + \vec{F}_{ANT_MR} + \vec{F}_{ANT_IR} + \vec{F}_{AG_SR} \quad (26)$$

$$J\Delta\ddot{\theta}_{HR} = \vec{T}_{HR_R_MF} + \vec{T}_{HR_L_MF} + \vec{T}_{Kp_HR} + \vec{T}_{Bp_HR} \quad (27)$$

$$\vec{T}_{VR_T_MF} = \vec{F}_{SR} + \vec{F}_{VR_T_MF} + \vec{F}_{AG_SR} \quad (28)$$

$$\vec{T}_{VR_B_MF} = \vec{F}_{MR} + \vec{F}_{IR} + \vec{F}_{LR} + \vec{F}_{VR_B_MF} + \vec{F}_{ANT_MR} + \vec{F}_{ANT_IR} + \vec{F}_{AG_LR} \quad (29)$$

$$J\Delta\ddot{\theta}_{VR} = \vec{T}_{VR_T_MF} + \vec{T}_{VR_B_MF} + \vec{T}_{Kp_VR} + \vec{T}_{Bp_VR} \quad (30)$$

$$\Delta\dot{\theta}_{HR} = \Delta\dot{\theta}_{HR} \quad (31)$$

$$\Delta\dot{\theta}_{VR} = \Delta\dot{\theta}_{VR} \quad (32)$$

$$\dot{F}_{HR_AG}(t) = \frac{N_{AG} - \hat{F}_{HR_AG}(t)}{\tau_{AG_sac}} \quad (33)$$

$$\dot{F}_{HR_ANT}(t) = \frac{N_{ANT} - \hat{F}_{HR_ANT}(t)}{\tau_{ANT_sac}} \quad (34)$$

$$\dot{F}_{VR_AG}(t) = \frac{N_{AG} - \hat{F}_{VR_AG}(t)}{\tau_{AG_sac}} \quad (35)$$

$$\dot{F}_{VR_ANT}(t) = \frac{N_{ANT} - \hat{F}_{VR_ANT}(t)}{\tau_{ANT_sac}} \quad (36)$$

4. Left Downward Eye Movement

4.1. Horizontal Left Muscle Force (HR_L_MF)

$$T_{HR_L_MF} = -F_{MR} \cos \theta_{MR} - K_{LT}(\theta_{HR_LT_MR} - \Delta\theta_{HR_LT_MR}) \cos \theta_{MR} + B_{AG} \Delta\dot{\theta}_{LT_MR} \cos \theta_{MR} \quad (37)$$

Resisting the contraction, the series elasticity components of medial rectus propagates the contractile force by pulling the eye globe with the same force $T_{HR_L_MF}$.

$$T_{HR_L_MF} = -K_{SE}(\theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}) \cos \theta_{MR} \quad (38)$$

Equations (37) and (38) can be used to calculate the force $T_{HR_L_MF}$ in terms of the eye rotation $\Delta\theta_{HR}$ and displacement $\Delta\theta_{HR_LT_MR}$ of the length tension component of the muscle.

$$\begin{aligned} -K_{SE}(\theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}) \cos \theta_{MR} + F_{MR} \cos \theta_{MR} + K_{LT}(\theta_{HR_LT_MR} - \Delta\theta_{HR_LT_MR}) \cos \theta_{MR} \\ - B_{AG} \Delta\dot{\theta}_{HR_LT_MR} \cos \theta_{MR} = 0 \end{aligned}$$

Taking into consideration that $\theta_{HR_LR} = \theta_{HR_LT_MR} + \theta_{HR_SE_MR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_LT_MR} - \Delta\theta_{HR_SE_MR}$

the following equation can be calculated:

$$\theta_{HR_MR} - \Delta\theta_{HR} = \theta_{HR_LT_MR} + \theta_{HR_SE_MR} - \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_SE_MR}$$

$$\theta_{HR_MR} - \Delta\theta_{HR} - \theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR} = \theta_{HR_SE_MR} + \Delta\theta_{HR_SE_MR}$$

$$-K_{SE}(\theta_{HR_MR} - \Delta\theta_{HR} - \theta_{HR_LT_MR} + \Delta\theta_{HR_LT_MR}) + F_{MR} + K_{LT}(\theta_{HR_LT_MR} - \Delta\theta_{HR_LT_MR}) - B_{AG} \Delta\dot{\theta}_{HR_LT_MR} = 0$$

$$\begin{aligned} -K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR}) + (F_{MR} - K_{SE}(\theta_{HR_MR} - \theta_{HR_LT_MR}) + K_{LT}\theta_{HR_LT_MR}) - K_{LT}\Delta\theta_{HR_LT_MR} \\ - B_{AG} \Delta\dot{\theta}_{HR_LT_MR} = 0 \end{aligned}$$

Assigning $\hat{F}_{MR} = F_{MR} - K_{SE}(\theta_{HR_MR} - \theta_{HR_LT_MR}) + K_{LT}\theta_{HR_LT_MR}$

$$-K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR}) = -\hat{F}_{MR} + K_{LT}\Delta\theta_{HR_LT_MR} + B_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

New equation for $T_{HR_L_MF}$ can be written as:

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR})$$

$$T_{HR_L_MF} = -\hat{F}_{MR} + K_{LT}\Delta\theta_{HR_LT_MR} + B_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

$$\Delta\theta_{HR} - \frac{T_{HR_L_MF}}{K_{SE}} = \Delta\theta_{HR_LT_MR}$$

$$T_{HR_L_MF} = -\hat{F}_{MR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{HR_L_MF}}{K_{SE}}\right) + B_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR}) \quad (39)$$

$$T_{HR_L_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_MR} \quad (40)$$

4.2. Horizontal Right Muscle Force (HR_R_MF)

$$T_{LR} = F_{LR} \cos \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) \cos \theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR} \cos \theta_{LR}$$

$$T_{IR} = F_{IR} \sin \theta_{IR} + K_{LT}(\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) \sin \theta_{IR} - B_{AG}\Delta\dot{\theta}_{VR_LT_IR} \sin \theta_{IR}$$

$$T_{SR} = F_{SR} \sin \theta_{SR} + K_{LT}(\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_SR}) \sin \theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_SR} \sin \theta_{SR}$$

$$T_{LR} = K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \cos \theta_{LR}$$

$$T_{IR} = K_{SE}(\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}) \sin \theta_{IR}$$

$$T_{SR} = K_{SE}(\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}) \sin \theta_{SR}$$

Above six equations can be used to calculate forces T_{LR}, T_{IR}, T_{SR} in terms of the eye rotation $\Delta\theta_{HR}$ $\Delta\theta_{VR}$ and displacements $\Delta\theta_{HR_LT_LR}, \Delta\theta_{VR_LT_IR}, \Delta\theta_{HR_LT_SR}$ of the length tension components of each muscle respectively.

4.2.1. Lateral Recuts Muscle Force (T_{LR}) of the Horiztonal Right Muscle Force

$$-K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \cos \theta_{LR} + F_{LR} \cos \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) \cos \theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR} \cos \theta_{LR} = 0$$

Taking into consideration that $\theta_{HR_LR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_SE_LR}$

the follwing equation can be calculated:

$$\theta_{HR_LR} + \Delta\theta_{HR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR} + \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_SE_LR}$$

$$\theta_{HR_LR} + \Delta\theta_{HR} - \theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR} = \theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}$$

$$-K_{SE}(\theta_{HR_LR} + \Delta\theta_{HR} - \theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR}) + F_{LR} + K_{LT}(\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR} = 0$$

$$-K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR}) + F_{LR} - K_{SE}(\theta_{HR_LR} - \theta_{HR_LT_LR}) + K_{LT}\theta_{HR_LT_LR} + K_{LT}\Delta\theta_{HR_LT_LR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR} = 0$$

Assigning $\hat{F}_{LR} = F_{LR} - K_{SE}(\theta_{HR_LR} - \theta_{HR_LT_LR}) + K_{LT}\theta_{HR_LT_LR}$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR}) = \hat{F}_{LR} + K_{LT}\Delta\theta_{HR_LT_LR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR}$$

New equation for T_{LR} can be written as:

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR})$$

$$\Delta\theta_{HR_LT_LR} = \Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}$$

$$T_{LR} = \hat{F}_{LR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}\right) + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{LR} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_LR}$$

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR})$$

4.2.2. Superior Recuts Muscle Force (T_{SR}) of the Horizontal Right Muscle Force

$$-K_{SE}(\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}) \sin \theta_{SR} + F_{SR} \sin \theta_{SR} + K_{LT}(\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_SR}) \sin \theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_LR} \sin \theta_{SR} = 0$$

Taking into consideration that $\theta_{VR_SR} = \theta_{VR_LT_SR} + \theta_{VR_SE_SR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR_LT_SR} + \Delta\theta_{VR_SE_SR}$

the following equation can be calculated:

$$\theta_{VR_SR} + \Delta\theta_{VR} = \theta_{VR_LT_SR} + \theta_{VR_SE_SR} + \Delta\theta_{VR_LT_SR} + \Delta\theta_{VR_SE_SR}$$

$$\theta_{VR_SR} + \Delta\theta_{VR} - \theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR} = \theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}$$

$$-K_{SE}(\theta_{VR_SR} + \Delta\theta_{VR} - \theta_{VR_LT_SR} - \Delta\theta_{VR_LT_SR}) + F_{SR} + K_{LT}(\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_SR}) + B_{ANT}\Delta\dot{\theta}_{VR_LT_LR} = 0$$

$$-K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR}) + F_{SR} - K_{SE}(\theta_{VR_SR} - \theta_{VR_LT_SR}) + K_{LT}\theta_{VR_LT_SR} + K_{LT}\Delta\theta_{VR_LT_SR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_SR} = 0$$

Assigning $\hat{F}_{SR} = F_{SR} - K_{SE}(\theta_{VR_SR} - \theta_{VR_LT_SR}) + K_{LT}\theta_{VR_LT_SR}$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR}) = \hat{F}_{SR} + K_{LT}\Delta\theta_{VR_LT_SR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_SR}$$

New equation for T_{SR} can be written as:

$$T_{SR} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR})$$

$$\Delta\theta_{VR_LT_SR} = \Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}$$

$$T_{SR} = S_{SR} + K_{LT} \left(\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}} \right) + B_{ANT}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{SR} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{SR} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR})$$

4.2.3. Inferior Recuts Muscle Force (T_{IR}) of the Horizontal Right Muscle Force

$$-K_{SE}(\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}) \sin \theta_{IR} + F_{IR} \sin \theta_{IR} + K_{LT}(\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) \sin \theta_{IR} - B_{AG}\Delta\dot{\theta}_{VR_LT_IR} \sin \theta_{IR} = 0$$

Taking into consideration that $\theta_{VR_IR} = \theta_{VR_LT_IR} + \theta_{VR_SE_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR_LT_IR} - \Delta\theta_{VR_SE_IR}$

the following equation can be calculated:

$$\theta_{VR_IR} - \Delta\theta_{VR} = \theta_{VR_LT_IR} + \theta_{VR_SE_IR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_SE_IR}$$

$$\theta_{VR_IR} - \Delta\theta_{VR} - \theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR} = \theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}$$

$$-K_{SE}(\theta_{VR_IR} - \Delta\theta_{VR} - \theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}) + F_{IR} + K_{LT}(\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) - B_{AG}\Delta\dot{\theta}_{VR_LT_IR} = 0$$

$$-K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) + F_{IR} - K_{SE}(\theta_{VR_IR} - \theta_{VR_LT_IR}) + K_{LT}\theta_{VR_LT_IR} - K_{LT}\Delta\theta_{VR_LT_IR} - B_{AG}\Delta\dot{\theta}_{VR_LT_IR} = 0$$

$$\text{Assigning } \hat{F}_{IR} = F_{IR} - K_{SE}(\theta_{VR_IR} - \theta_{VR_LT_IR}) + K_{LT}\theta_{VR_LT_IR}$$

$$K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) = \hat{F}_{IR} - K_{LT}\Delta\theta_{VR_LT_IR} - B_{AG}\Delta\dot{\theta}_{VR_LT_IR}$$

New equation for T_{IR} can be written as:

$$T_{IR} = K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR})$$

$$\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR} = \Delta\theta_{VR_LT_IR}$$

$$T_{IR} = \hat{F}_{IR} - K_{LT}\left(\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR}\right) - B_{AG}\Delta\dot{\theta}_{VR_LT_IR}$$

$$T_{IR} = \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_IR}$$

$$T_{IR} = K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR})$$

4.2.4. Formulating Horizontal RightMuscle Force ($T_{HR_R_MF}$)

$$T_{HR_R_MF} = T_{LR} + T_{IR} + T_{SR}$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR}) + K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) + K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR})$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_SR})$$

$$T_{HR_R_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_LR} + \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_IR} + \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{HR_R_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_LR} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_IR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_SR}$$

$$T_{HR_R_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_LR} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_IR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_SR} \quad (41)$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_SR}) \quad (42)$$

4.3. Vertical Bottom Muscle Force ($T_{VR_B_MF}$)

$$T_{VR_B_MF} = -F_{IR} \cos \theta_{IR} - K_{LT}(\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) \cos \theta_{IR} + B_{AG}\Delta\dot{\theta}_{VR_LT_IR} \cos \theta_{IR} \quad (43)$$

Resisting the contraction, the series elasticity components of medial rectus propagates the contractile force by pulling the eye globe with the same force $T_{HR_L_MF}$.

$$T_{VR_B_MF} = -K_{SE}(\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}) \cos \theta_{IR} \quad (44)$$

Equations (43) and (44) can be used to calculate the force $T_{VR_B_MF}$ in terms of the eye rotation $\Delta\theta_{VR}$ and displacement $\Delta\theta_{VR_LT_IR}$ of the length tension component of the muscle.

$$\begin{aligned} K_{SE}(\theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}) \cos \theta_{IR} - F_{IR} \cos \theta_{IR} - K_{LT}(\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) \cos \theta_{IR} + B_{AG}\Delta\dot{\theta}_{VR_LT_IR} \cos \theta_{IR} \\ = 0 \end{aligned}$$

Taking into consideration that $\theta_{VR_IR} = \theta_{VR_LT_IR} + \theta_{VR_SE_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR_LT_IR} - \Delta\theta_{VR_SE_IR}$

the following equation can be calculated:

$$\theta_{VR_IR} - \Delta\theta_{VR} = \theta_{VR_LT_IR} + \theta_{VR_SE_IR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_SE_IR}$$

$$\theta_{VR_IR} - \Delta\theta_{VR} - \theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR} = \theta_{VR_SE_IR} + \Delta\theta_{VR_SE_IR}$$

$$K_{SE}(\theta_{VR_IR} - \Delta\theta_{VR} - \theta_{VR_LT_IR} + \Delta\theta_{VR_LT_IR}) - F_{IR} - K_{LT}(\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_IR}) + B_{AG}\Delta\dot{\theta}_{VR_LT_IR} = 0$$

$$K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) - F_{IR} + K_{SE}(\theta_{VR_IR} - \theta_{VR_LT_IR}) - K_{LT}\theta_{VR_LT_IR} + K_{LT}\Delta\theta_{VR_LT_IR} + B_{AG}\Delta\dot{\theta}_{VR_LT_IR} = 0$$

$$\text{Assigning } \hat{F}_{IR} = F_{IR} + K_{SE}(\theta_{VR_LT_IR} - \theta_{VR_IR}) + K_{LT}\theta_{VR_LT_IR}$$

$$-K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) = -\hat{F}_{IR} + K_{LT}\Delta\theta_{VR_LT_IR} + B_{AG}\Delta\dot{\theta}_{VR_LT_IR}$$

New equation for $T_{VR_B_MF}$ can be written as:

$$T_{VR_B_MF} = -K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR})$$

$$\Delta\theta_{VR} - \frac{T_{VR_B_MF}}{K_{SE}} = \Delta\theta_{VR_LT_IR}$$

$$T_{VR_B_MF} = -\hat{F}_{IR} + K_{LT} \left(\Delta\theta_{VR} - \frac{T_{VR_B_MF}}{K_{SE}} \right) + B_{AG}\Delta\dot{\theta}_{VR_LT_IR}$$

$$T_{VR_B_MF} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_IR} \quad (45)$$

$$T_{VR_B_MF} = -K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) \quad (46)$$

4.4. Vertical Top Muscle Force ($T_{VR_T_MF}$)

$$T_{LR} = F_{LR} \sin \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) \sin \theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR} \sin \theta_{LR}$$

$$T_{MR} = F_{MR} \sin \theta_{MR} + K_{LT}(\theta_{HR_LT_MR} - \Delta\theta_{HR_LT_MR}) \sin \theta_{MR} - B_{AG}\Delta\dot{\theta}_{HR_LT_MR} \sin \theta_{MR}$$

$$T_{SR} = F_{SR} \cos \theta_{SR} + K_{LT}(\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_SR}) \cos \theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR_LT_SR} \cos \theta_{SR}$$

$$T_{LR} = K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \sin \theta_{LR}$$

$$T_{MR} = K_{SE}(\theta_{MR_SE_MR} + \Delta\theta_{MR_SE_MR}) \sin \theta_{MR}$$

$$T_{SR} = K_{SE}(\theta_{VR_SE_SR} + \Delta\theta_{VR_SE_SR}) \cos \theta_{SR}$$

Above six equations can be used to calculate forces T_{LR} , T_{MR} , T_{SR} in terms of the eye rotation $\Delta\theta_{HR}$, $\Delta\theta_{VR}$ and displacements $\Delta\theta_{HR_LT_LR}$, $\Delta\theta_{VR_LT_MR}$, $\Delta\theta_{HR_LT_SR}$ of the length tension components of each muscle respectively.

4.4.1. Lateral Rectus Muscle Force (T_{LR}) of the Vertical Top Muscle Force

$$-K_{SE}(\theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}) \sin \theta_{LR} + F_{LR} \sin \theta_{LR} + K_{LT}(\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_LR}) \sin \theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR_LT_LR} \sin \theta_{LR} = 0$$

Taking into consideration that $\theta_{HR_LR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_SE_LR}$

the following equation can be calculated:

$$\theta_{HR_LR} + \Delta\theta_{HR} = \theta_{HR_LT_LR} + \theta_{HR_SE_LR} + \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_SE_LR}$$

$$\theta_{HR_LR} + \Delta\theta_{HR} - \theta_{HR_LT_LR} - \Delta\theta_{HR_LT_LR} = \theta_{HR_SE_LR} + \Delta\theta_{HR_SE_LR}$$

$$-K_{SE}(\theta_{HR,LR} + \Delta\theta_{HR} - \theta_{HR,LT,LR} - \Delta\theta_{HR,LT,LR}) + F_{LR} + K_{LT}(\theta_{HR,LT,LR} + \Delta\theta_{HR,LT,LR}) + B_{ANT}\Delta\dot{\theta}_{HR,LT,LR} = 0$$

$$-K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR,LT,LR}) + F_{LR} - K_{SE}(\theta_{HR,LR} - \theta_{HR,LT,LR}) + K_{LT}\theta_{HR,LT,LR} + K_{LT}\Delta\theta_{HR,LT,LR} + B_{ANT}\Delta\dot{\theta}_{HR,LT,LR} = 0$$

Assigning $\hat{F}_{LR} = F_{LR} - K_{SE}(\theta_{HR,LR} - \theta_{HR,LT,LR}) + K_{LT}\theta_{HR,LT,LR}$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR,LT,LR}) = \hat{F}_{LR} + K_{LT}\Delta\theta_{HR,LT,LR} + B_{ANT}\Delta\dot{\theta}_{HR,LT,LR}$$

New equation for T_{LR} can be written as:

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR,LT,LR})$$

$$\Delta\theta_{HR,LT,LR} = \Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}$$

$$T_{LR} = \hat{F}_{LR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}\right) + B_{ANT}\Delta\dot{\theta}_{HR,LT,LR}$$

$$T_{LR} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR,LT,LR}$$

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR,LT,LR})$$

4.4.2. Superior Recuts Muscle Force (T_{SR}) of the Vertical Top Muscle Force

$$-K_{SE}(\theta_{VR,SE,SR} + \Delta\theta_{VR,SE,SR}) \cos \theta_{SR} + F_{SR} \cos \theta_{SR} + K_{LT}(\theta_{VR,LT,SR} + \Delta\theta_{VR,LT,SR}) \cos \theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR,LT,LR} \cos \theta_{SR} = 0$$

Taking into consideration that $\theta_{VR,SR} = \theta_{VR,LT,SR} + \theta_{VR,SE,SR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR,LT,SR} + \Delta\theta_{VR,SE,SR}$

the following equation can be calculated:

$$\theta_{VR,SR} + \Delta\theta_{VR} = \theta_{VR,LT,SR} + \theta_{VR,SE,SR} + \Delta\theta_{VR,LT,SR} + \Delta\theta_{VR,SE,SR}$$

$$\theta_{VR,SR} + \Delta\theta_{VR} - \theta_{VR,LT,SR} - \Delta\theta_{VR,LT,SR} = \theta_{VR,SE,SR} + \Delta\theta_{VR,SE,SR}$$

$$-K_{SE}(\theta_{VR,SR} + \Delta\theta_{VR} - \theta_{VR,LT,SR} - \Delta\theta_{VR,LT,SR}) + F_{SR} + K_{LT}(\theta_{VR,LT,SR} + \Delta\theta_{VR,LT,SR}) + B_{ANT}\Delta\dot{\theta}_{VR,LT,LR} = 0$$

$$-K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_{LT_{SR}}}) + F_{SR} - K_{SE}(\theta_{VR_{SR}} - \theta_{VR_{LT_{SR}}}) + K_{LT}\theta_{VR_{LT_{SR}}} + K_{LT}\Delta\theta_{VR_{LT_{SR}}} + B_{ANT}\Delta\dot{\theta}_{VR_{LT_{SR}}} = 0$$

Assigning $\hat{F}_{SR} = F_{SR} - K_{SE}(\theta_{VR_{SR}} - \theta_{VR_{LT_{SR}}}) + K_{LT}\theta_{VR_{LT_{SR}}}$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_{LT_{SR}}}) = \hat{F}_{SR} + K_{LT}\Delta\theta_{VR_{LT_{SR}}} + B_{ANT}\Delta\dot{\theta}_{VR_{LT_{SR}}}$$

New equation for T_{LR} can be written as:

$$T_{SR} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_{LT_{SR}}})$$

$$\Delta\theta_{VR_{LT_{SR}}} = \Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}$$

$$T_{SR} = \hat{F}_{SR} + K_{LT}\left(\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}\right) + B_{ANT}\Delta\dot{\theta}_{VR_{LT_{SR}}}$$

$$T_{SR} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_{LT_{SR}}}$$

$$T_{SR} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_{LT_{SR}}})$$

4.4.3. Medial Recuts Muscle Force (T_{MR}) of the Vertical Top Muscle Force

$$-K_{SE}(\theta_{HR_{SE_{MR}}} + \Delta\theta_{HR_{SE_{MR}}}) \sin \theta_{MR} + F_{MR} \sin \theta_{MR} + K_{LT}(\theta_{HR_{LT_{MR}}} - \Delta\theta_{VR_{LT_{MR}}}) \sin \theta_{MR} - B_{AG}\Delta\dot{\theta}_{HR_{LT_{MR}}} \sin \theta_{MR} = 0$$

Taking into consideration that $\theta_{HR_{MR}} = \theta_{HR_{LT_{MR}}} + \theta_{HR_{SE_{MR}}}$ and $\Delta\theta_{HR} = \Delta\theta_{HR_{LT_{MR}}} - \Delta\theta_{HR_{SE_{MR}}}$

the following equation can be calculated:

$$\theta_{HR_{MR}} - \Delta\theta_{HR} = \theta_{HR_{LT_{MR}}} + \theta_{HR_{SE_{MR}}} - \Delta\theta_{HR_{LT_{MR}}} + \Delta\theta_{HR_{SE_{MR}}}$$

$$\theta_{HR_{MR}} - \Delta\theta_{HR} - \theta_{HR_{LT_{MR}}} + \Delta\theta_{HR_{LT_{MR}}} = \theta_{HR_{SE_{MR}}} + \Delta\theta_{HR_{SE_{MR}}}$$

$$-K_{SE}(\theta_{HR_{MR}} - \Delta\theta_{HR} - \theta_{HR_{LT_{MR}}} + \Delta\theta_{HR_{LT_{MR}}}) + F_{MR} + K_{LT}(\theta_{HR_{LT_{MR}}} - \Delta\theta_{HR_{LT_{MR}}}) - B_{AG}\Delta\dot{\theta}_{HR_{LT_{MR}}} = 0$$

$$\begin{aligned}
 & -K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR}) + (F_{MR} - K_{SE}(\theta_{HR_MR} - \theta_{HR_LT_MR}) + K_{LT}\theta_{HR_LT_MR}) - K_{LT}\Delta\theta_{HR_LT_MR} \\
 & - B_{AG}\Delta\dot{\theta}_{HR_LT_MR} = 0
 \end{aligned}$$

Assigning $\hat{F}_{MR} = F_{MR} - K_{SE}(\theta_{HR_MR} - \theta_{HR_LT_MR}) + K_{LT}\theta_{HR_LT_MR}$

$$K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR}) = \hat{F}_{MR} - K_{LT}\Delta\theta_{HR_LT_MR} - B_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

New equation for $T_{HR_L_MF}$ can be written as:

$$T_{MR} = K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR})$$

$$T_{MR} = \hat{F}_{MR} - K_{LT}\Delta\theta_{HR_LT_MR} - B_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

$$\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR_LT_MR}$$

$$T_{MR} = \hat{F}_{MR} - K_{LT} \left(\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} \right) - B_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{MR} = \frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

$$T_{MR} = K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR})$$

4.4.4. Formulating Vertical Top Muscle Force ($T_{VR_T_MF}$)

$$T_{VR_T_MF} = T_{LR} + T_{MR} + T_{SR}$$

$$\begin{aligned}
 T_{VR_T_MF} = & \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR_LT_LR} + \frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_MR} + \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} \\
 & + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR_LT_SR}
 \end{aligned}$$

$$T_{VR_T_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{MR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR} - \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_MR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR} \quad (47)$$

$$T_{VR_T_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR}) + K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR}) + K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR})$$

$$T_{VR_T_MF} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR} + \Delta\theta_{HR_LT_MR} - \Delta\theta_{HR_LT_LR}) \quad (48)$$

5. Muscle Model Equations

5.1. Right Upward Eye Movement Equations

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR_{LT_{LR}}} - \Delta\theta_{HR}) \quad (49)$$

$$T_{HR_R_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_{LT_{LR}}} \quad (50)$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_{LT_{MR}}} - \Delta\theta_{VR_{LT_{IR}}} + \Delta\theta_{VR_{LT_{SR}}}) \quad (51)$$

$$T_{HR_L_MF} = -\frac{(\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{SR})K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_{LT_{MR}}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_{LT_{IR}}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_{LT_{SR}}} \quad (52)$$

$$T_{VR_T_MF} = K_{SE}(\Delta\theta_{VR_{LT_{SR}}} - \Delta\theta_{VR}) \quad (53)$$

$$T_{VR_T_MF} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_{LT_{SR}}} \quad (54)$$

$$T_{VR_B_MF} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_{LT_{IR}}} - \Delta\theta_{HR_{LT_{MR}}} + \Delta\theta_{HR_{LT_{LR}}}) \quad (55)$$

$$T_{VR_B_MF} = -\left(\frac{\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{LR}}{K_{SE} + K_{LT}}\right)K_{SE} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR_{LT_{MR}}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR_{LT_{IR}}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR_{LT_{LR}}} \quad (56)$$

5.2. Left Downward Eye Movement Equations

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR_{LT_{MR}}} - \Delta\theta_{HR}) \quad (57)$$

$$T_{HR_L_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR_{LT_{MR}}} \quad (58)$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{VR_LT_IR} - \Delta\theta_{VR_LT_SR}) \quad (59)$$

$$T_{HR_R_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} + \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR} - \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_IR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR} \quad (60)$$

$$T_{VR_B_MF} = -K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR}) \quad (61)$$

$$T_{VR_B_MF} = -\frac{\hat{F}_{IR} K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_IR} \quad (62)$$

$$T_{VR_T_MF} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR} + \Delta\theta_{HR_LT_MR} - \Delta\theta_{HR_LT_LR}) \quad (63)$$

$$T_{VR_T_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{MR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR} - \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_MR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR} \quad (64)$$

5.3. Muscle Model properties

With the use of above eye movements, we found feature properties in each 8 equations of movements; Right Upward and Left Downward, to develop muscle model equations. These feature properties are used to build muscle model equations, which can be used to calculate 8 equations in all four types of eye movements: Right Upward, Right Downward, Left Upward, and Left Downward.

Eye Movement	Agonist	Antagonist
Right Upward	Lateral Rectus Superior Rectus	Medial Rectus Inferior Rectus
Left Downward	Medial Rectus Inferior Rectus	Lateral Rectus Superior Rectus

In order to calculate 8 force equations for a particular eye movement, muscle model properties needs to be identified. In our model we found following model properties in each of the eye movement to successfully predict force equations.

- **Plane** – Horizontal (HR) or Vertical (VR)
- **ForceDirection** – Top (T), Bottom (B), Right (R), or Left (L)
- **AgonistMuscles** – Agonist Muscles of the eye movement model (muscles that contribute to ForceDirection)
- **AntagonistMuscles** – Antagonist Muscles of the eye movement model (muscles that contribute to ForceDirection)
- **Sign** – For this research, following ForceDirection sign convention was used. ForceDirections Right (R), Top (T) as positive and ForceDirections Left (L), Bottom (B) as negative.

Above properties can be easily extracted with the use of a muscle model diagram. Formation of muscle model equations is explained in the following.

5.3.1. Formulating Muscle Model Equations

- Equations (49), (53), (57), and (61) show a similar pattern in its formation, by changing its sign based on its ForceDirection. Also, displacements in length tension due to horizontal (HR) or vertical (VR) eye movement is only based on AgonistMuscles of that particular plane (HR, or VR). This information is used to formulate following muscle model equation.

Muscle Model 1

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

- Equations (50), (54), (58) and (62) show a similar pattern in its formation, by changing its sign based on its ForceDirection. Also the active state tension of the agonist muscle always takes the sign of the ForceDirection and other two components takes the opposite sign of the ForceDirection. This information is used to formulate following muscle model equation.

$$\text{Muscle Model 2} \quad T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle} K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

- Equations (51) , (55) , (59) and (63) show a similar pattern in its formation, by changing its sign based on its ForceDirection. Also, sign of the displacements in Series elasticity is same as the sign of the ForceDirection. Furthermore, the displacement in length tension of the AgonistMuscle is always positive and the displacement in length tension of the AntagonistMuscle is always negative. They both take a plane perpendicular to the ForceDirection plane. Also, displacement in the length tension component of the plane always an Antagonist muscle which takes negative sign for it. This information is used to formulate following muscle model equation.

$$\begin{aligned} \text{Muscle Model 3} \quad T_{plane_ForceDirection_MF} &= \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} \\ &\quad - \Delta\theta_{perpendicularPlane_LT_AntagonistMuscle} \\ &\quad + \Delta\theta_{perpendicularPlane_LT_AgonistMuscle}) \end{aligned}$$

- Equations (52) , (56), (60) and (64) show a similar pattern in its formation, by changing its sign based on its ForceDirection. Also, collective values of the active state tensions

take the similar sign as the ForceDirection. Active state tensions are consists of single agonist and two antagonist muscles. Damping component of the agonist muscle always takes the opposite sign of the ForceDirection. But Damping components of the antagonist muscles always take the sign similar to the ForceDirection. Damping components of the antagonist muscles are consists of one from the same plane as the ForceDirection and other in a perpendicular plane. Displacement of the Series elasticity component takes the same sign as ForceDirection. This information is used to formulate following muscle model equation.

Muscle Model 4

$$\begin{aligned}
 T_{plane_ForceDirection_MF} &= \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \\
 &\pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \\
 &\pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \\
 &\pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle}
 \end{aligned}$$

5.3.2. Criteria to build 8 Muscle Equations using Muscle Model Equations

- Identify Muscle Model properties using the Muscle Model Diagram, given that antagonist muscles are defined as those that are pulling the eye globe and the antagonist muscles as those that are being pulled.
- Following muscle force equations need to be identified
 - Horizontal Left Muscle Force (2 equations)
 - Horizontal Right Muscle Force (2 equations)
 - Vertical Top Muscle Force (2 equations)
 - Vertical Bottom Muscle Force (2 equations)

- With the use of identified Model Properties, apply model equations in the following manner
 - If there are no force components in this ForceDirection consider AntagonistMuscles=NULL. When this is the case use model equation 1 and 2 to calculate force equations
 - Else use model equation 3 and 4 to calculate rest of the force equations
- Following plane, muscle relationship needs to be used throughout the force equation calculation
 - If a particular equation consists of plane_LT_AgonistMuscle or plane_LT_AntagonistMuscle or PerpendicularPlane_LT_AgonistMuscle or PerpendicularPlane_LT_AntagonistMuscle, then muscle should be in the plane specified by the relationship.

Example:

Plane=HR ForceDirection=R AgonistMuscles= LR AntagonistMuscles=NULL Sign=positive

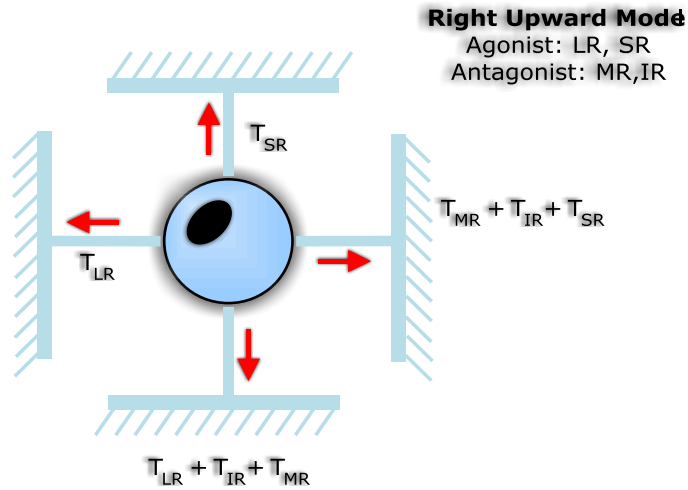
$$T_{plane_direction_MF} = \pm K_{SE}(\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})$$

- Muscle should be in the plane specified in the muscle model properties.
- Following sign convention needs to be used in appropriate muscle equations. For this research, following ForceDirection sign convention was used. ForceDirections Right (R), Top (T) as positive and ForceDirections Left (L), Bottom (B) as negative.

5.4. Eye movements: Right Upward, Right Downward, Left Upward, and Left Downward using Muscle Model Equations

5.4.1. Right Upward Eye Movement



5.4.1.1. Horizontal Right Muscle Force

Plane	HR
ForceDirection	R
AgonistMuscles	LR
AntagonistMuscles	NULL
Sign	positive

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE}(\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{HR_R_MF} = K_{SE}(\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle}K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane}K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{HR_R_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

5.4.1.2. Vertical Top Muscle Force

Plane	VR
ForceDirection	T
AgonistMuscles	SR
AntagonistMuscles	NULL
Sign	positive

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{VR_T_MF} = K_{SE} (\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle} K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{VR_T_MF} = \frac{\hat{F}_{SR} K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_SR}$$

5.4.1.3. Horizontal Left Muscle Force

Plane	HR
ForceDirection	L
AgonistMuscles	SR
AntagonistMuscles	MR,IR
Sign	negative

By considering muscle model 3:

$$\begin{aligned} T_{plane_ForceDirection_MF} &= \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{perpendicularPlane_LT_AntagonistMuscle} \\ &\quad + \Delta\theta_{perpendicularPlane_LT_AgonistMuscle}) \end{aligned}$$

$$T_{HR_L_MF} = -K_{SE} (\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR} - \Delta\theta_{VR_LT_IR} + \Delta\theta_{VR_LT_SR})$$

By considering muscle model 4:

$$\begin{aligned} T_{plane_ForceDirection_MF} &= \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle}) K_{SE}}{K_{SE} + K_{LT}} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \\ &\quad \mp \hat{B}_{AG} \Delta\dot{\theta}_{perpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \\ &\quad \pm \hat{B}_{ANT} \Delta\dot{\theta}_{perpendicularPlane_LT_AntagonistMuscle} \end{aligned}$$

$$T_{HR_L_MF} = -\frac{(\hat{F}_{SR} + \hat{F}_{MR} + \hat{F}_{IR})}{K_{SE} + K_{LT}} K_{SE} - \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_SR} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR}$$

5.4.1.4. Vertical Bottom Muscle Force

Plane	VR
ForceDirection	B
AgonistMuscles	LR
AntagonistMuscles	MR,IR
Sign	negative

By considering muscle model 3:

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{PerpendicularPlane_LT_AntagonistMuscle} + \Delta\theta_{PerpendicularPlane_LT_AgonistMuscle})$$

$$T_{VR_B_MF} = -K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_IR} - \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_LR})$$

By considering muscle model 4:

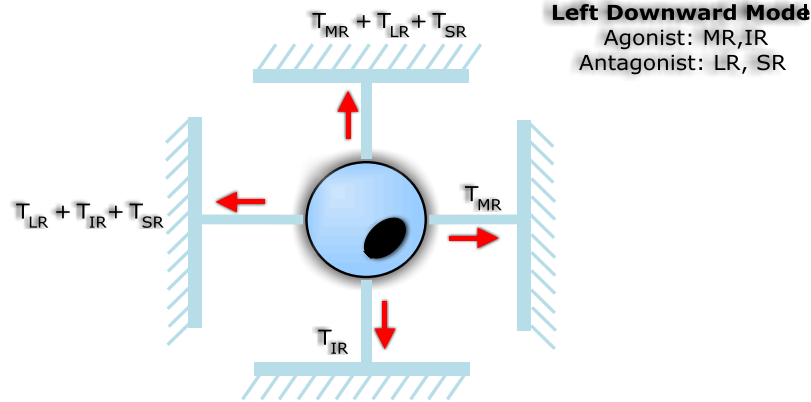
$$T_{plane_ForceDirection_MF} = \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}}$$

$$\mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle}$$

$$\pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle}$$

$$T_{VR_B_MF} = -\frac{(\hat{F}_{LR} + \hat{F}_{IR} + \hat{F}_{MR})}{K_{SE} + K_{LT}} K_{SE} - \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR}$$

5.4.2. Left Downward Eye Movement



5.4.2.1. Horizontal Left Muscle Force

Plane	HR
ForceDirection	L
AgonistMuscles	MR
AntagonistMuscles	NULL
Sign	negative

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE}(\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle}K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane}K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{HR_L_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

5.4.2.2. Vertical Bottom Muscle Force

Plane	VR
ForceDirection	B
AgonistMuscles	IR
AntagonistMuscles	NULL
Sign	negative

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{VR_B_MF} = -K_{SE} (\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle} K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{VR_B_MF} = -\frac{\hat{F}_{IR} K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_IR}$$

5.4.2.3. Horizontal Right Muscle Force

Plane	HR
ForceDirection	R
AgonistMuscles	IR
AntagonistMuscles	LR,SR
Sign	positive

By considering muscle model 3:

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{PerpendicularPlane_LT_AntagonistMuscle} + \Delta\theta_{PerpendicularPlane_LT_AgonistMuscle})$$

$$T_{HR_R_MF} = K_{SE} (\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR} - \Delta\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_IR})$$

By considering muscle model 4:

$$T_{plane_ForceDirection_MF} = \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle}) K_{SE}}{K_{SE} + K_{LT}} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle}$$

$$T_{HR_R_MF} = \frac{(\hat{F}_{IR} + \hat{F}_{LR} + \hat{F}_{SR}) K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_IR} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR}$$

5.4.2.4. Vertical Top Muscle Force

Plane	VR
ForceDirection	T
AgonistMuscles	MR
AntagonistMuscles	LR, SR
Sign	positive

By considering muscle model 3:

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{PerpendicularPlane_LT_AntagonistMuscle} + \Delta\theta_{PerpendicularPlane_LT_AgonistMuscle})$$

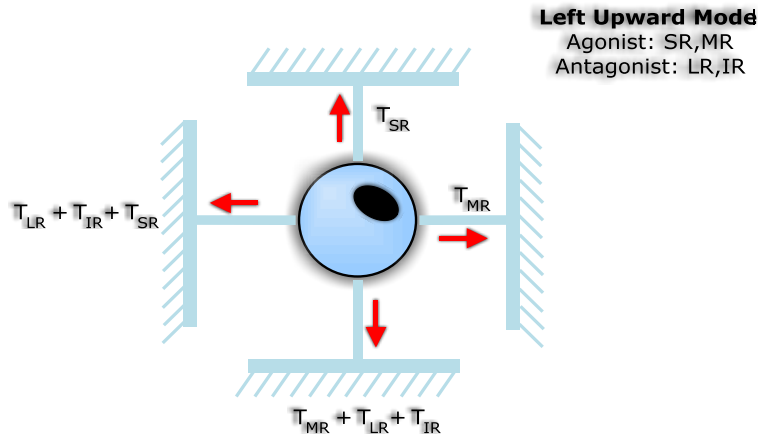
$$T_{VR_T_MF} = K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_MR})$$

By considering muscle model 4:

$$T_{plane_ForceDirection_MF} = \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle}$$

$$T_{VR_T_MF} = \frac{(\hat{F}_{MR} + \hat{F}_{LR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_MR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR}$$

5.4.3. Left Upward Eye Movement



5.4.3.1. Horizontal Left Muscle Force

Plane	HR
ForceDirection	L
AgonistMuscles	MR
AntagonistMuscles	NULL
Sign	negative

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE}(\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR_LT_MR} - \Delta\theta_{HR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle}K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane}K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{HR_L_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_MR}$$

5.4.3.2. Vertical Top Muscle Force

Plane	VR
ForceDirection	T
AgonistMuscles	SR
AntagonistMuscles	NULL
Sign	positive

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE}(\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{VR_T_MF} = K_{SE}(\Delta\theta_{VR_LT_SR} - \Delta\theta_{VR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle}K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane}K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{VR_T_MF} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_SR}$$

5.4.3.3. Horizontal Right Muscle Force

Plane	HR
ForceDirection	R
AgonistMuscles	IR
AntagonistMuscles	LR, SR
Sign	positive

By considering muscle model 3:

$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{perpendicularPlane_LT_AntagonistMuscle} + \Delta\theta_{perpendicularPlane_LT_AgonistMuscle})$$

$$T_{HR_R_MF} = K_{SE} (\Delta\theta_{HR} - \Delta\theta_{HR_LT_LR} - \Delta\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_JR})$$

By considering muscle model 4:

$$T_{plane_ForceDirection_MF} = \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{perpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{perpendicularPlane_LT_AntagonistMuscle}$$

$$T_{HR_R_MF} = \frac{(\hat{F}_{IR} + \hat{F}_{LR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} + \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_JR} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR}$$

5.4.3.4. Vertical Bottom Muscle Force

Plane	VR
ForceDirection	B
AgonistMuscles	MR
AntagonistMuscles	LR, IR
Sign	negative

By considering muscle model 3:

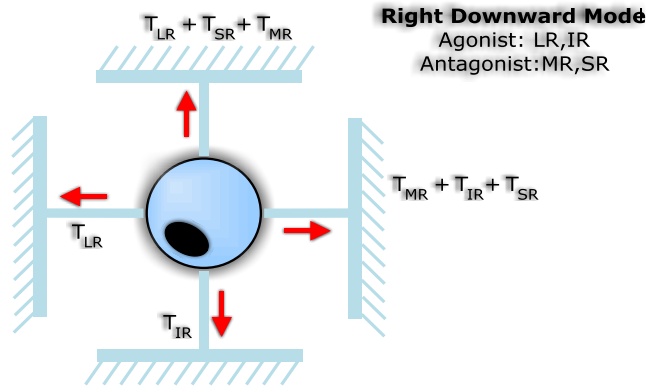
$$T_{plane_ForceDirection_MF} = \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{perpendicularPlane_LT_AntagonistMuscle} + \Delta\theta_{perpendicularPlane_LT_AgonistMuscle})$$

$$T_{VR_B_MF} = -K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_JR} - \Delta\theta_{HR_LT_LR} + \Delta\theta_{HR_LT_MR})$$

By considering muscle model 4:

$$\begin{aligned}
 T_{plane_ForceDirection_MF} &= \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \\
 &\mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \\
 &\pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle} \\
 T_{VR_B_MF} &= -\frac{(\hat{F}_{MR} + \hat{F}_{LR} + \hat{F}_{IR})}{K_{SE} + K_{LT}} K_{SE} - \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_IR} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_LR}
 \end{aligned}$$

5.4.4. Right Downward Eye Movement



5.4.4.1. Horizontal Right Muscle Force

Plane	HR
ForceDirection	R
AgonistMuscles	LR
AntagonistMuscles	NULL
Sign	positive

By considering muscle model 1:

$$\begin{aligned}
 T_{plane_ForceDirection_MF} &= \pm K_{SE} (\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane}) \\
 T_{HR_R_MF} &= K_{SE} (\Delta\theta_{HR_LT_LR} - \Delta\theta_{HR})
 \end{aligned}$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle} K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG} \Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{HR_R_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR_LT_LR}$$

5.4.4.2. Vertical Bottom Muscle Force

Plane	VR
ForceDirection	B
AgonistMuscles	IR
AntagonistMuscles	NULL
Sign	negative

By considering muscle model 1:

$$T_{plane_ForceDirection_MF} = \pm K_{SE}(\Delta\theta_{plane_LT_AgonistMuscle} - \Delta\theta_{plane})$$

$$T_{VR_B_MF} = -K_{SE}(\Delta\theta_{VR_LT_IR} - \Delta\theta_{VR})$$

By considering muscle model 2:

$$T_{plane_ForceDirection_MF} = \pm \frac{\hat{F}_{AgonistMuscle}K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{plane}K_{SE}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{plane_LT_AgonistMuscle}$$

$$T_{VR_B_MF} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR_LT_IR}$$

5.4.4.3. Horizontal Left Muscle Force

Plane	HR
ForceDirection	L
AgonistMuscles	IR
AntagonistMuscles	MR, SR
Sign	negative

By considering muscle model 3:

$$T_{plane_ForceDirection_MF}$$

$$= \pm K_{SE}(\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{PerpendicularPlane_LT_AntagonistMuscle} + \Delta\theta_{PerpendicularPlane_LT_AgonistMuscle})$$

$$T_{HR_L_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR_LT_MR} - \Delta\theta_{VR_LT_SR} + \Delta\theta_{VR_LT_IR})$$

By considering muscle model 4:

$$\begin{aligned}
 T_{plane_ForceDirection_MF} &= \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \\
 &\mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \\
 &\pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle} \\
 T_{HR_L_MF} &= -\frac{(\hat{F}_{IR} + \hat{F}_{MR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} - \frac{\Delta\theta_{HR} K_{SE}}{K_{SE} + K_{LT}} + \hat{B}_{AG} \Delta\dot{\theta}_{VR_LT_IR} - \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR} - \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR}
 \end{aligned}$$

5.4.4.4. Vertical Top Muscle Force

Plane	VR
ForceDirection	T
AgonistMuscles	LR
AntagonistMuscles	MR, SR
Sign	positive

By considering muscle model 3:

$$\begin{aligned}
 T_{plane_ForceDirection_MF} &= \pm K_{SE} (\Delta\theta_{plane} - \Delta\theta_{plane_LT_AntagonistMuscle} - \Delta\theta_{PerpendicularPlane_LT_AntagonistMuscle} \\
 &\quad + \Delta\theta_{PerpendicularPlane_LT_AgonistMuscle}) \\
 T_{VR_T_MF} &= K_{SE} (\Delta\theta_{VR} - \Delta\theta_{VR_LT_SR} - \Delta\theta_{HR_LT_MR} + \Delta\theta_{HR_LT_LR})
 \end{aligned}$$

By considering muscle model 4:

$$\begin{aligned}
 T_{plane_ForceDirection_MF} &= \pm \frac{(\hat{F}_{AgonistMuscle} + \hat{F}_{AntagonistMuscle} + \hat{F}_{AntagonistMuscle})}{K_{SE} + K_{LT}} K_{SE} \pm \frac{\Delta\theta_{plane} K_{SE}}{K_{SE} + K_{LT}} \\
 &\mp \hat{B}_{AG} \Delta\dot{\theta}_{PerpendicularPlane_LT_AgonistMuscle} \pm \hat{B}_{ANT} \Delta\dot{\theta}_{plane_LT_AntagonistMuscle} \\
 &\pm \hat{B}_{ANT} \Delta\dot{\theta}_{PerpendicularPlane_LT_AntagonistMuscle} \\
 T_{VR_T_MF} &= \frac{(\hat{F}_{LR} + \hat{F}_{MR} + \hat{F}_{SR})}{K_{SE} + K_{LT}} K_{SE} + \frac{\Delta\theta_{VR} K_{SE}}{K_{SE} + K_{LT}} - \hat{B}_{AG} \Delta\dot{\theta}_{HR_LT_LR} + \hat{B}_{ANT} \Delta\dot{\theta}_{VR_LT_SR} + \hat{B}_{ANT} \Delta\dot{\theta}_{HR_LT_MR}
 \end{aligned}$$