Two-Dimensional Linear Homeomorphic Oculomotor Plant Mathematical Model Equations

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1. Oculomotor Plant Mathematical Model (OPMM)

The oculomotor plant can be represented by a mechanical model composed of six muscles attached to the eye globe, as shown in Figures 1 and 2. The model presented in this paper considers only four of these muscles – the lateral, medial, superior, and inferior recti – along with the eye globe and surrounding tissue.

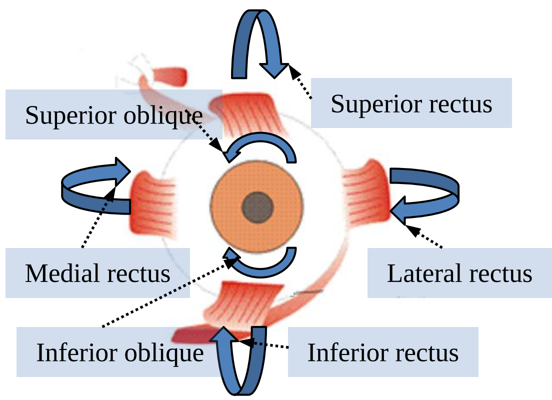


Figure 1. Left eye diagram (front view).

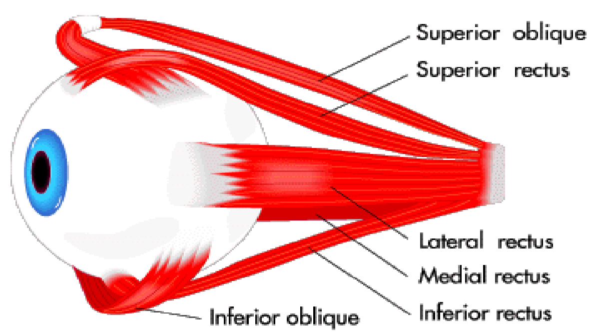


Figure 2. Left eye diagram (side view).

The eye globe has a radius of approximately 11 millimeters, and the lateral, medial, superior, and inferior recti are modeled through a system of mechanical components described in the following sections. Each muscle can play the role of agonist (AG) or antagonist (ANT), depending on the direction of movement. The agonist muscle pulls the eye globe in the required direction, while the antagonist muscle resists the pull. As an example, when the right eye moves in the right-upward direction, the lateral and superior recti play the role of agonist and support the movement, while the medial and inferior recti play the role of antagonist and resist the movement. Each muscle/role combination is considered in its own section. The eye globe can move in eight directions within the two-dimensional plane: right, left, upward, downward, right-upward, left-upward, right-downward, and left-downward. This paper considers only right-upward and left-downward movements of the right eye in detail, as this provides all relevant equations necessary for eye movement simulation in the two-dimensional plane. The distinction between right/left eye equations is important, as the term lateral rectus refers to the outer muscle (closest to the ear), while the term medial rectus refers to the inner muscle (closest to the nose). Therefore, when considering the left eye, the labels referring to the lateral/medial recti must be swapped.

2. Two-Dimensional Oculomotor Plant Mathematical Model (2DOPMM)

The OPMM consists of four extraocular muscles which provide the necessary force to rotate the eye globe; these include the lateral rectus (LR), superior rectus (SR), medial rectus (MR), and inferior rectus (IR). In the equations and figures presented in this paper subscript notation will identify parameters that belong to each muscle, parameters without subscripts are assumed to be identical for the relevant muscles. Figure 3 illustrates the eye globe held in the central coordinate position (0, 0), where opposing muscles apply equal force to maintain a stable fixation. When the eye moves to a particular position from the central coordinate position, each muscle connected to the eye globe contracts or stretches accordingly, as shown in Figure 4.

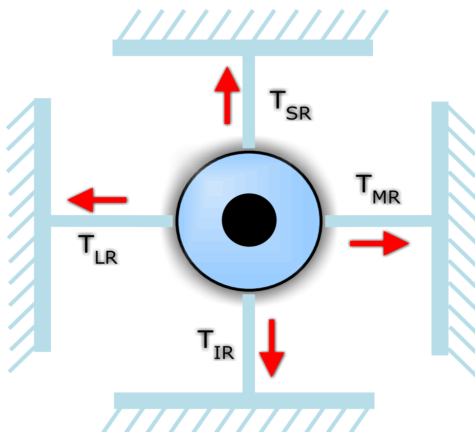


Figure 3. OPMM with four muscle forces.

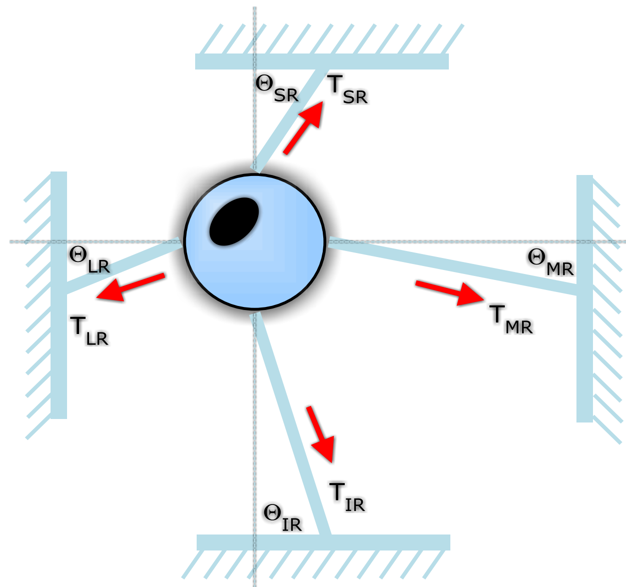


Figure 4. Right-upward movement of the right eye.

Consider a right-upward movement of the right eye, which directs the eye's visual axis from the central coordinate position: the lateral and superior recti supply the force necessary to move the eye globe to the required position and maintain a stable fixation on the target, while the medial and inferior recti stretch and resist the movement. When the eye fixates on the target, the muscles compensate the forces and stabilize the eye globe. The contraction/stretching of each muscle cause the horizontal (HR) and vertical (VR) movement of the eye globe from its origin. Θ_{HR} and Θ_{VR} describe the angle of rotation with respect to each of the muscles connected to the eye globe, where $\Theta_{HR} = \Theta_{LR} + \Theta_{MR}$ and $\Theta_{VR} = \Theta_{SR} + \Theta_{IR}$.

Each muscle is innervated to contract/stretch by a neuronal control signal, causing a projection of its forces at its point of connection to the eye globe. The projection of each muscle force is directed according to the direction of movement, allowing four basic muscle force equations, as shown in Figure 5.

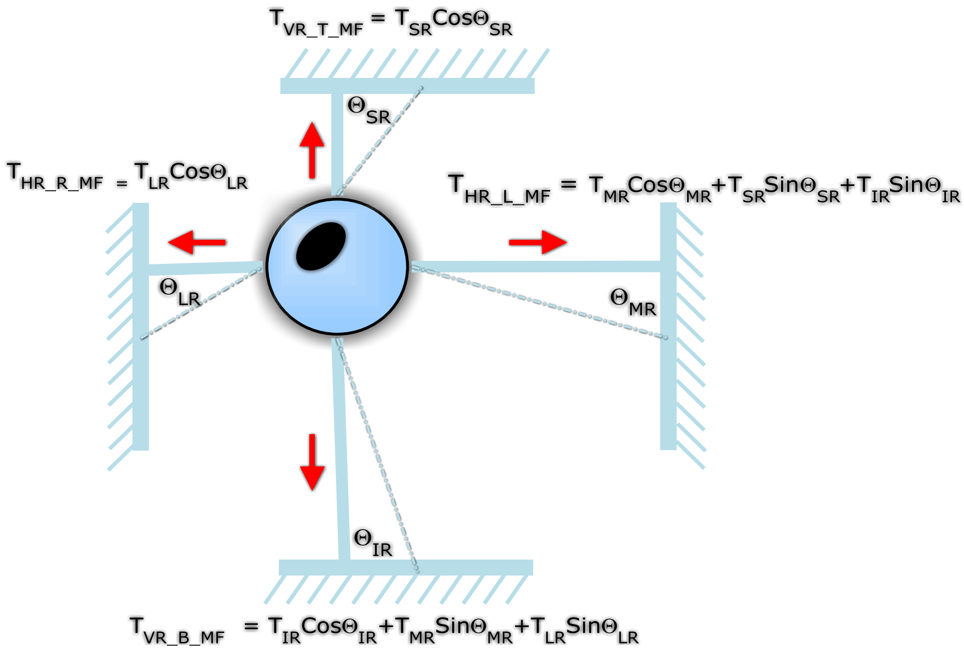


Figure 5. Right-upward movement with horizontally and vertically projected muscle forces.

3. Right-upward Movement

The muscle mechanical model (MMM) of the lateral rectus is shown in Figure 6. The neuronal control signal N_{LR} creates the active-state tension $\vec{F}_{HR\_LR}$ that works in parallel with the length-tension force $\vec{F}_{HR\_LT\_LR}$. Together they produce tension $\vec{T}_{HR\_R\_MF} = \vec{F}_{HR\_LR} + \vec{F}_{HR\_LT\_LR}$ that is propagated through the series elasticity to the eye globe: $\vec{T}_{HR\_R\_MF} = \vec{F}_{HR\_SE\_LR}$.

Scalar values of the forces are as follows: the length-tension force of the lateral rectus is $F_{HR\_LT\_LR} = K_{LT}\theta_{HR\_LT\_LR}$, where $\theta_{HR\_LT\_LR}$ is the displacement of the length-tension component in the horizontal direction and K_{LT} is the spring coefficient; force propagated by the series elasticity component is $T_{LR} = K_{SE}\theta_{HR\_SE\_LR}$, where $\theta_{HR\_SE\_LR}$ is the displacement of the series elasticity component in the horizontal direction and K_{SE} is the spring coefficient.

Tension $\vec{T}_{LR}$ applied by the lateral rectus to the eye globe is counterbalanced by the tension of the medial, superior, and inferior recti ($\vec{T}_{MR} + \vec{T}_{SR} + \vec{T}_{IR}$) and tension created by the passive elasticity of the muscles and tissue surrounding the eye globe $\overrightarrow{K_P\Delta\theta}$. The MMM becomes more complex during eye rotations, making it necessary to present MMMs for the horizontal and vertical muscle forces separately. To provide more detail, muscle forces are described with scalar values.

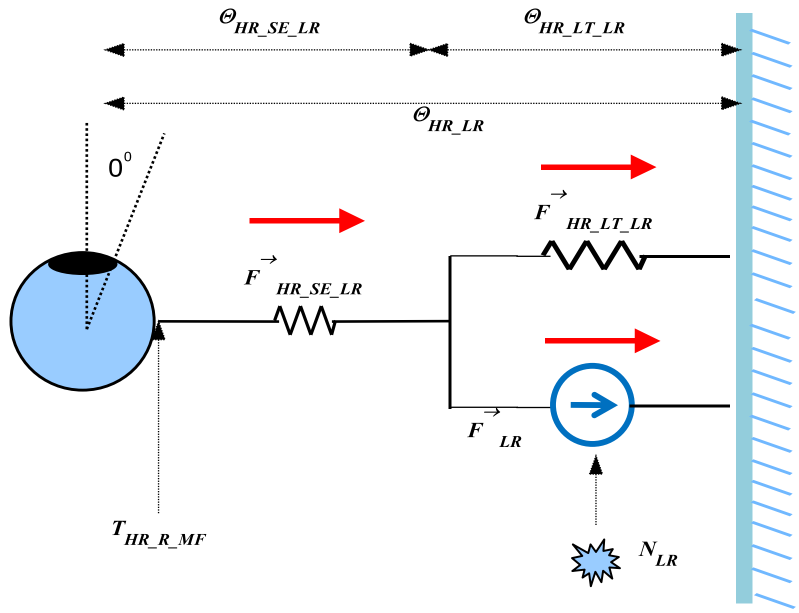


Figure 6. Muscle mechanical model of the lateral rectus.

3.1. Horizontal Right Muscle Force (HR_R_MF)

The agonist muscle contracts, rotates the eye globe, and stretches the antagonist muscle. Assuming the lateral rectus plays the role of agonist, Figure 7 presents the MMM of the horizontal right muscle force pulling the eye globe in the positive direction.

Prior to the eye movement, the displacement in the series elasticity and length-tension components of the lateral rectus is θ_{HR_LR}. When the eye moves to the right by $\Delta\theta_{HR}$ degrees, the original displacement θ_{HR_LR} is reduced, making the resulting displacement $\theta_{HR\_LR} - \Delta\theta_{HR}$. The displacement $\Delta\theta_{HR}$ can be broken into the displacement of its components: $\Delta\theta_{HR} = \Delta\theta_{HR\_SE\_LR} - \Delta\theta_{HR\_LT\_LR}$. Muscle contraction expands the series elasticity and shortens the length-tension components, making the resulting displacement $\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}$ and $\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}$ respectively. The force-velocity relationship represented by the damping component $B_{AG}\Delta\dot{\theta}_{LT\_LR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based on the velocity of contraction of the length-tension component.

Using Figure 7, we can write the equation of force generated by contraction of the lateral rectus:

$$T_{HR\_R\_MF} = F_{LR} \cos\theta_{LR} + K_{LT}(\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}) \cos\theta_{LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \cos\theta_{LR} \quad \textbf{(1)}$$

Resisting the contraction, the series elasticity component propagates the contractile force by pulling the eye globe with the same force:

$$T_{HR\_R\_MF} = K_{SE}(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}) \cos\theta_{LR} \quad \textbf{(2)}$$

Equations (1) and (2) can then be used to calculate the force $T_{HR\_R\_MF}$ in terms of the eye rotation $\Delta\theta_{HR}$ and displacement $\Delta\theta_{HR\_LT\_LR}$ of the length-tension component:

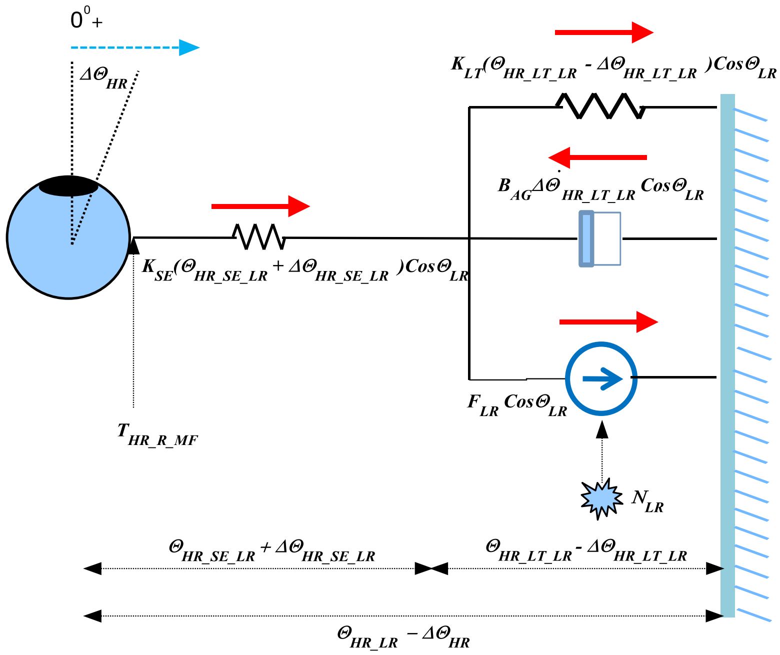


Figure 7. Horizontal right muscle force mechanical model.

$$-K_{SE}(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}) \cos\theta_{LR} + F_{LR}\cos\theta_{LR} + K_{LT}(\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}) \cos\theta_{LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}\cos\theta_{LR} = 0$$

Taking into consideration that $\theta_{HR\_LR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_SE\_LR}$, the following equations can be derived:

$$\theta_{HR\_LR} - \Delta\theta_{HR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR} - \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$\theta_{HR\_LR} - \Delta\theta_{HR} - \theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR} = \theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$-K_{SE}\left(\theta_{HR\_LR} - \Delta\theta_{HR} - \theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR}\right) + F_{LR} + K_{LT}\left(\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}\right) - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

$$-K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) + \left(F_{LR} - K_{SE}\left(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}\right) + K_{LT}\theta_{HR\_LT\_LR}\right) - K_{LT}\Delta\theta_{HR\_LT\_LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

Assigning:

$$\hat{F}_{LR} = F_{LR} - K_{SE}\left(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}\right) + K_{LT}\theta_{HR\_LT\_LR}$$

$$K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) = \hat{F}_{LR} - K_{LT}\Delta\theta_{HR\_LT\_LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

The new equation for $T_{HR\_R\_MF}$ can be written as:

$$T_{HR\_R\_MF} = K_{SE}(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR})$$

$$T_{HR\_R\_MF} = \hat{F}_{LR} - K_{LT}\Delta\theta_{HR\_LT\_LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$\frac{T_{HR\_R\_MF}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR\_LT\_LR}$$

$$T_{HR\_R\_MF} = \hat{F}_{LR} - K_{LT}\left(\frac{T_{HR\_R\_MF}}{K_{SE}} + \Delta\theta_{HR}\right) - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{HR\_R\_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \quad \textbf{(3)}$$

$$T_{HR\_R\_MF} = K_{SE}(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}) \quad \textbf{(4)}$$

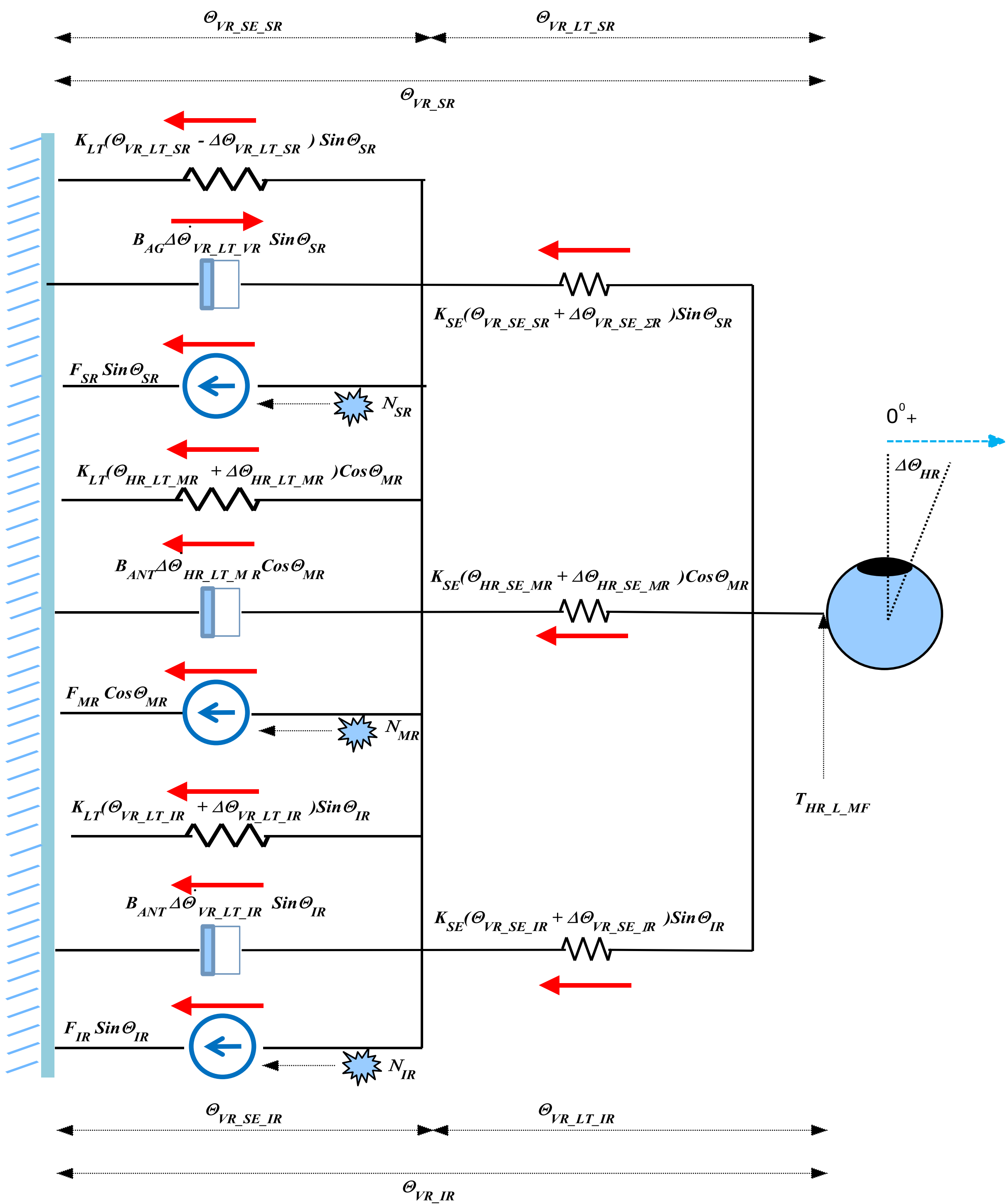


Figure 8. Horizontal left muscle force mechanical model.

3.2. Horizontal Left Muscle Force (HR_L_MF)

The antagonist muscle is stretched by the agonist pull. Assuming the medial and inferior recti play the role of antagonist, Figure 8 presents the MMM of the horizontal left muscle force resisting the pull on the eye globe in the positive direction.

Prior to the eye movement, the displacement in the series elasticity and length-tension components of the medial rectus is θ_{HR_MR}. When the eye moves to the right by $\Delta\theta_{HR}$ degrees, the original displacement θ_{HR_MR} is increased, making the resulting displacement $\theta_{HR\_MR} + \Delta\theta_{HR}$. The displacement $\Delta\theta_{HR}$ can be broken into the displacement of its components: $\Delta\theta_{HR} = \Delta\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_LT\_MR}$. Muscle contraction expands the series elasticity and length-tension components, making the resulting displacement $\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}$ and $\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}$ respectively. The force-velocity relationship represented by the damping component $B_{ANT}\Delta\dot{\theta}_{LT\_MR}$ resists the muscle stretching. The amount of resistive force produced by the damping component is based on the velocity of stretching of the length-tension component.

The displacement in the series elasticity and length-tension components of the superior rectus is θ_{VR_SR}. When the eye moves upward by $\Delta\theta_{VR}$ degrees, the original displacement θ_{VR_SR} is reduced, making the resulting displacement $\theta_{VR\_SR} - \Delta\theta_{VR}$. The displacement $\Delta\theta_{VR}$ can be broken into the displacement of its components: $\Delta\theta_{VR} = \Delta\theta_{VR\_SE\_SR} - \Delta\theta_{VR\_LT\_SR}$. Muscle contraction expands the series elasticity and shortens the length-tension components, making the resulting displacement $\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR}$ and $\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR}$ respectively. The force-velocity relationship represented by the damping component $B_{AG}\Delta\dot{\theta}_{LT\_SR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based on the velocity of contraction of the length-tension component.

The displacement in the series elasticity and length-tension components of the inferior rectus is θ_{VR_IR}. When the eye moves upward by $\Delta\theta_{VR}$ degrees, the original displacement θ_{VR_IR} is increased, making the resulting displacement $\theta_{VR\_IR} + \Delta\theta_{VR}$. The displacement $\Delta\theta_{VR}$ can be broken into the displacement of its components: $\Delta\theta_{VR} = \Delta\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_LT\_IR}$. Muscle contraction expands the series elasticity and length-tension components, making the resulting displacement $\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}$ and $\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}$ respectively. The force-velocity relationship represented by the damping component $B_{ANT}\Delta\dot{\theta}_{LT\_IR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based on the velocity of contraction of the length-tension component.

We can write the equations of force generated by contraction/stretching of the extraocular muscles as follows:

$$T_{MR} = -F_{MR} \cos\theta_{MR} - K_{LT}(\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}) \cos\theta_{MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} \cos\theta_{MR}$$

$$T_{IR} = -F_{IR} \sin\theta_{IR} - K_{LT}(\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}) \sin\theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} \sin\theta_{IR}$$

$$T_{SR} = -F_{SR} \sin\theta_{SR} - K_{LT}(\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR}) \sin\theta_{SR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} \sin\theta_{SR}$$

$$T_{MR} = -K_{SE}(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}) \cos\theta_{MR}$$

$$T_{IR} = -K_{SE}(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}) \sin\theta_{IR}$$

$T_{SR} = -K_{SE}(\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR}) \sin\theta_{SR}$

These equations can be used to calculate the forces T_{MR}, T_{IR}, and T_{SR} in terms of the eye rotation, $\Delta\theta_{HR}$ and $\Delta\theta_{VR}$, and displacement of the length-tension component of each muscle, $\Delta\theta_{HR\_LT\_MR}$, $\Delta\theta_{VR\_LT\_IR}$, and $\Delta\theta_{VR\_LT\_SR}$.

3.2.1. *Medial Rectus Tension (T_{MR}) of the Horizontal Left Muscle Force*

$$\boldsymbol{K_{SE}(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}) \cos\theta_{MR} - F_{MR} \cos\theta_{MR} - K_{LT}(\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}) \cos\theta_{MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} \cos\theta_{MR} = 0}$$

Taking into consideration that $\theta_{HR\_MR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_SE\_MR}$, the following equations can be derived:

$$\theta_{HR\_MR} + \Delta\theta_{HR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR} + \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$\theta_{HR\_MR} + \Delta\theta_{HR} - \theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR} = \theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$K_{SE}(\theta_{HR\_MR} + \Delta\theta_{HR} - \theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR}) - F_{MR} - K_{LT}(\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}) - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}) - F_{MR} - K_{SE}\theta_{HR\_LT\_MR} + K_{SE}\theta_{HR\_MR} - K_{LT}\theta_{HR\_LT\_MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

Assigning:

$$\hat{F}_{MR} = F_{MR} + K_{SE}\theta_{HR\_LT\_MR} - K_{SE}\theta_{HR\_MR} + K_{LT}\theta_{HR\_LT\_MR}$$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}) = \hat{F}_{MR} + K_{LT}\Delta\theta_{HR\_LT\_MR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$-K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}) = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

The new equation for T_{MR} can be written as:

$$T_{MR} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR})$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR}$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\left(\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR}\right) - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{MR} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{MR} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR})$$

3.2.2. *Inferior Rectus Tension (T_{IR}) of the Horizontal Left Muscle Force*

$$K_{SE}(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}) \sin\theta_{IR} - F_{IR} \sin\theta_{IR} - K_{LT}(\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}) \sin\theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} \sin\theta_{IR} = 0$$

Taking into consideration that $\theta_{VR\_IR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_SE\_IR}$, the following equations can be derived:

$$\theta_{VR\_IR} + \Delta\theta_{VR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR} + \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_SE\_IR}$$

$$\theta_{VR\_IR} + \Delta\theta_{VR} - \theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR} = \theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}$$

$$K_{SE}(\theta_{VR\_IR} + \Delta\theta_{VR} - \theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR}) - F_{IR} - K_{LT}(\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}) - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} = 0$$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}) - F_{IR} - K_{SE}\theta_{VR\_LT\_IR} + K_{SE}\theta_{VR\_IR} - K_{LT}\theta_{VR\_LT\_IR} - K_{LT}\Delta\theta_{VR\_LT\_IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} = 0$$

Assigning:

$$\hat{F}_{IR} = F_{IR} + K_{SE}\theta_{VR\_LT\_IR} - K_{SE}\theta_{VR\_IR} + K_{LT}\theta_{VR\_LT\_IR}$$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}) = \hat{F}_{IR} + K_{LT}\Delta\theta_{VR\_LT\_IR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$-K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}) = -\hat{F}_{IR} - K_{LT}\Delta\theta_{VR\_LT\_IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

The new equation for T_{IR} can be written as:

$$T_{IR} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR})$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\Delta\theta_{VR\_LT\_IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR} = \Delta\theta_{VR\_LT\_IR}$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\left(\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR}\right) - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{IR} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{IR} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR})$$

3.2.3. Superior Rectus Tension (T_{SR}) of the Horizontal Left Muscle Force

$$-K_{SE}(\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR})\sin\theta_{SR} + F_{SR}\sin\theta_{SR} + K_{LT}(\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR})\sin\theta_{SR} - B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}\sin\theta_{SR} = 0$$

Taking into consideration that $\theta_{VR\_SR} = \theta_{VR\_LT\_SR} + \theta_{VR\_SE\_SR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_SE\_SR}$, the following equations can be derived:

$$\theta_{VR\_SR} - \Delta\theta_{VR} = \theta_{VR\_LT\_SR} + \theta_{VR\_SE\_SR} - \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_SE\_SR}$$

$$\theta_{VR\_SR} - \Delta\theta_{VR} - \theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_SR} = \theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR}$$

$$-K_{SE}(\theta_{VR\_SR} - \Delta\theta_{VR} - \theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_SR}) + F_{SR} + K_{LT}(\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR}) - B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} = 0$$

$$-K_{SE}(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}) + \left(F_{SR} - K_{SE}(\theta_{VR\_SR} - \theta_{VR\_LT\_SR}) + K_{LT}\theta_{VR\_LT\_SR}\right) - K_{LT}\Delta\theta_{VR\_LT\_SR} - B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} = 0$$

Assigning:

$$\hat{F}_{SR} = F_{SR} - K_{SE}(\theta_{VR\_SR} - \theta_{VR\_LT\_SR}) + K_{LT}\theta_{VR\_LT\_SR}$$

$$K_{SE}(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}) = \hat{F}_{SR} - K_{LT}\Delta\theta_{VR\_LT\_SR} - B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$-K_{SE}(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}) = -\hat{F}_{SR} + K_{LT}\Delta\theta_{VR\_LT\_SR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

The new equation for T_{SR} can be written as:

$$T_{SR} = -K_{SE}(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR})$$

$$T_{SR} = -\hat{F}_{SR} + K_{LT}\Delta\theta_{VR\_LT\_SR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}} = \Delta\theta_{VR\_LT\_SR}$$

$$T_{SR} = -\hat{F}_{SR} + K_{LT}\left(\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}\right) + B_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{SR} = -\frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{SR} = -K_{SE}(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR})$$

3.2.4. Formulating Horizontal Left Muscle Force ($T_{HR\_L\_MF}$)

$$T_{HR\_L\_MF} = T_{MR} + T_{IR} + T_{SR}$$

$$T_{HR\_L\_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \frac{\hat{F}_{IR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} - \frac{\hat{F}_{SR}K_{SE}}{K_{SE}+K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{HR\_L\_MF} = -\frac{\left(\hat{F}_{MR}+\hat{F}_{IR}+\hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_MR} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{HR\_L\_MF} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right) - K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}\right) - K_{SE}\left(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}\right)$$

$$T_{HR\_L\_MF} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right) + K_{SE}\Delta\theta_{VR\_LT\_IR} - K_{SE}\Delta\theta_{VR\_LT\_SR}$$

$$T_{HR\_L\_MF} = -K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_SR}) \quad \textbf{(5)}$$

$$T_{HR\_L\_MF} = -\frac{\left(\hat{F}_{MR}+\hat{F}_{IR}+\hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(6)}$$

3.3. Vertical Top Muscle Force (VR_T_MF)

The agonist muscle contracts, rotates the eye globe, and stretches the antagonist muscle. Assuming the superior rectus plays the role of agonist, Figure 9 presents the MMM of the vertical top muscle force pulling the eye globe in the positive direction.

Prior to the eye movement, the displacement in the series elasticity and length-tension components of the superior rectus is θ_{VR_SR}. When the eye moves upward by $\Delta\theta_{VR}$ degrees, the original displacement θ_{VR_SR} is reduced, making the resulting displacement $\theta_{VR\_SR} - \Delta\theta_{VR}$. The displacement $\Delta\theta_{VR}$ can be broken into the displacement of its components: $\Delta\theta_{VR} = \Delta\theta_{VR\_SE\_SR} - \Delta\theta_{VR\_LT\_SR}$. Muscle contraction expands the series elasticity and shortens the length-tension components, making the resulting displacement $\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR}$ and $\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR}$ respectively. The force-velocity relationship represented by the damping component $B_{AG}\Delta\dot{\theta}_{LT\_SR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based on the velocity of contraction of the length-tension component.

We can write the equation of force generated by contraction of the superior rectus:

$$T_{VR\_T\_SR} = F_{SR}\cos\theta_{SR} + K_{LT}(\theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR})\cos\theta_{SR} - B_{AG}\Delta\dot{\theta}_{LT\_SR}\cos\theta_{SR} \quad \textbf{(7)}$$

Resisting the contraction, the series elasticity component propagates the contractile force by pulling the eye globe with the same force:

$$T_{VR\_T\_MF} = K_{SE}(\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR})\cos\theta_{SR} \quad \textbf{(8)}$$

Equations (5) and (6) can be used to calculate the force $T_{VR\_T\_MF}$ in terms of the eye rotation $\Delta\theta_{VR}$ and displacement $\Delta\theta_{VR\_LT\_SR}$ of the length-tension component.

$$T_{VR\_T\_MF} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{LT\_SR} \quad \textbf{(9)}$$

$$T_{VR\_T\_MF} = K_{SE}(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}) \quad \textbf{(10)}$$

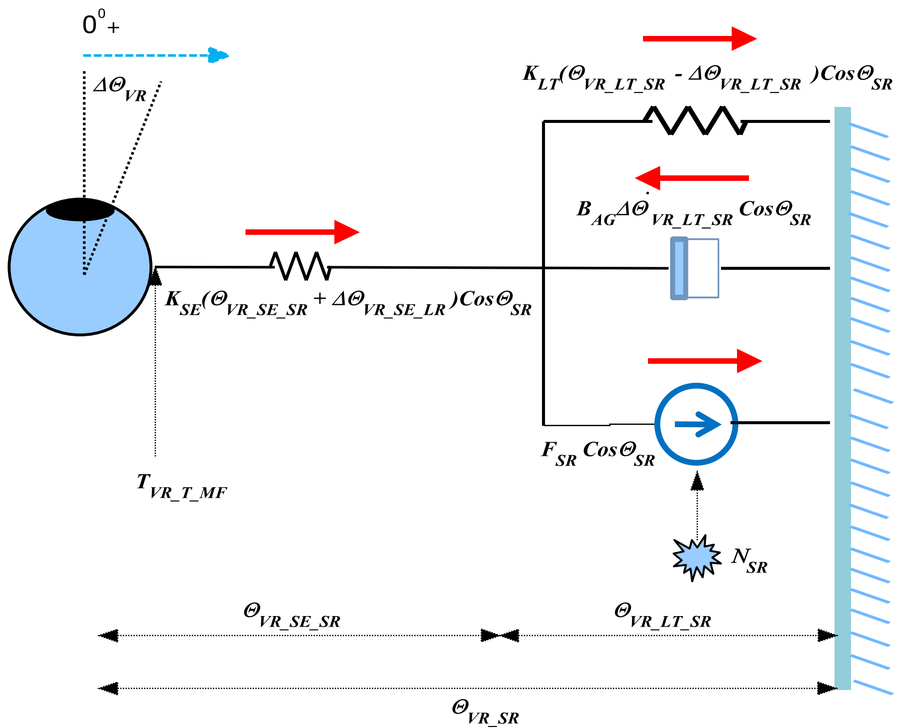


Figure 9. Vertical top muscle force mechanical model.

3.4. Vertical Bottom Muscle Force (VR_B_MF)

The antagonist muscle is stretched by the agonist pull. Assuming the medial and inferior recti play the role of antagonist, Figure 10 presents the MMM of the vertical bottom muscle force resisting the pull on the eye globe in the positive direction.

Prior to the eye movement, the displacement in the series elasticity and length-tension components of the inferior rectus is θ_{VR_IR}. When the eye moves upward by $\Delta\theta_{VR}$ degrees, the original displacement θ_{VR_IR} is increased, making the resulting displacement $\theta_{VR\_IR} + \Delta\theta_{VR}$. The displacement $\Delta\theta_{VR}$ can be broken into the displacement of its components: $\Delta\theta_{VR} = \Delta\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_LT\_IR}$. Muscle contraction expands the series elasticity and length-tension components, making the resulting displacement $\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}$ and $\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}$ respectively. The force-velocity relationship represented by the damping component $B_{ANT}\Delta\dot{\theta}_{LT\_IR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based on the velocity of contraction of the length-tension component.

The displacement in the series elasticity and length-tension components of the lateral rectus is θ_{HR_LR}. When the eye moves to the right by $\Delta\theta_{HR}$ degrees, the original displacement θ_{HR_LR} is reduced, making the resulting displacement $\theta_{HR\_LR} - \Delta\theta_{HR}$. The displacement $\Delta\theta_{HR}$ can be broken into the displacement of its components: $\Delta\theta_{HR} = \Delta\theta_{HR\_SE\_LR} - \Delta\theta_{HR\_LT\_LR}$. Muscle contraction expands the series elasticity and shortens the length-tension components, making the resulting displacement $\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}$ and $\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}$ respectively. The force-velocity relationship represented by the damping component $B_{AG}\Delta\dot{\theta}_{LT\_LR}$ resists the muscle contraction. The amount of resistive force produced by the damping component is based on the velocity of contraction of the length-tension component.

The displacement in the series elasticity and length-tension components of the medial rectus is θ_{HR_MR}. When the eye moves to the right by $\Delta\theta_{HR}$ degrees, the original displacement θ_{HR_MR} is increased, making the resulting displacement $\theta_{HR\_MR} + \Delta\theta_{HR}$. The displacement $\Delta\theta_{HR}$ can be broken into the displacement of its components: $\Delta\theta_{HR} = \Delta\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_LT\_MR}$. Muscle contraction expands the series elasticity and length-tension components, making the resulting displacement $\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}$ and $\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}$ respectively. The force-velocity relationship represented by the damping component $B_{ANT}\Delta\dot{\theta}_{LT\_MR}$ resists the muscle stretching. The amount of resistive force produced by the damping component is based on the velocity of stretching of the length-tension component.

We can write the equations of force generated by contraction/stretching of the extraocular muscles as follows:

$$T_{IR} = -F_{IR}\cos\theta_{IR} - K_{LT}\left(\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}\right)\cos\theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}\cos\theta_{IR}$$

$$T_{LR} = -F_{LR}\sin\theta_{LR} - K_{LT}\left(\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}\right)\sin\theta_{LR} + B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}\sin\theta_{LR}$$

$$T_{MR} = -F_{MR}\sin\theta_{MR} - K_{LT}\left(\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}\right)\sin\theta_{MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}\sin\theta_{MR}$$

$$T_{IR} = -K_{SE}\left(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}\right)\cos\theta_{IR}$$

$$T_{LR} = -K_{SE}\left(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}\right)\sin\theta_{LR}$$

$$T_{MR} = -K_{SE}\left(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}\right)\sin\theta_{MR}$$

These equations can be used to calculate the forces T_{IR}, T_{LR}, and T_{MR} in terms of the eye rotation, $\Delta\theta_{HR}$ and $\Delta\theta_{VR}$, and displacement of the length-tension component of each muscle, $\Delta\theta_{VR\_LT\_IR}$, $\Delta\theta_{HR\_LT\_LR}$, and $\Delta\theta_{HR\_LT\_MR}$.

3.4.1. Inferior Rectus Muscle Force (T_{IR}) of the Vertical Bottom Muscle Force

$$\boldsymbol{K_{SE}\left(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}\right)\cos\theta_{IR} - F_{IR}\cos\theta_{IR} - K_{LT}\left(\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}\right)\cos\theta_{IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}\cos\theta_{IR} = 0}$$

Taking into consideration that $\theta_{VR\_IR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_SE\_IR}$, the following equations can be derived:

$$\theta_{VR\_IR} + \Delta\theta_{VR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR} + \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_SE\_IR}$$

$$\theta_{VR\_IR} + \Delta\theta_{VR} - \theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR} = \theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}$$

$$K_{SE}\left(\theta_{VR\_IR} + \Delta\theta_{VR} - \theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR}\right) - F_{IR} - K_{LT}\left(\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}\right) - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} = 0$$

$$K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}\right) - F_{IR} - K_{SE}\theta_{VR\_LT\_IR} + K_{SE}\theta_{VR\_IR} - K_{LT}\theta_{VR\_LT\_IR} - K_{LT}\Delta\theta_{VR\_LT\_IR} - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} = 0$$

Assigning:

$$\hat{F}_{IR} = F_{IR} + K_{SE}\theta_{VR\_LT\_IR} - K_{SE}\theta_{VR\_IR} + K_{LT}\theta_{VR\_LT\_IR}$$

$$K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}\right) = \hat{F}_{IR} + K_{LT}\Delta\theta_{VR\_LT\_IR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

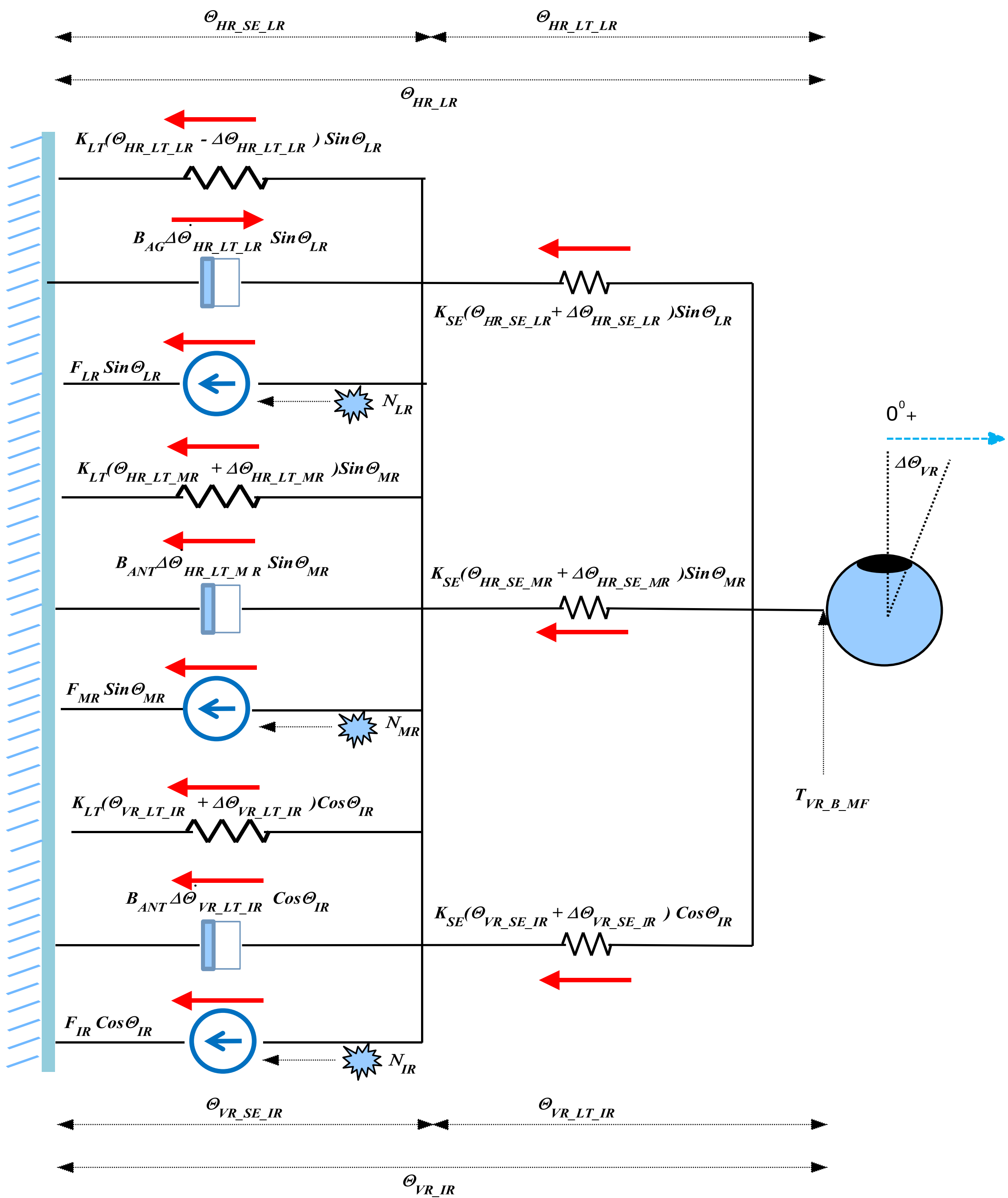


Figure 10. Vertical bottom muscle force mechanical model.

$$-K_{SE}\left(\Delta\theta_{VR}-\Delta\theta_{VR\_LT\_IR}\right)=-\hat{F}_{IR}-K_{LT}\Delta\theta_{VR\_LT\_IR}-B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

The new equation for T_{IR} can be written as:

$$T_{IR}=-K_{SE}\left(\Delta\theta_{VR}-\Delta\theta_{VR\_LT\_IR}\right)$$

$$T_{IR}=-\hat{F}_{IR}-K_{LT}\Delta\theta_{VR\_LT\_IR}-B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR} = \Delta\theta_{VR\_LT\_IR}$$

$$T_{IR} = -\hat{F}_{IR} - K_{LT}\left(\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR}\right) - B_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{IR} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{IR} = -K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}\right)$$

3.4.2. *Lateral Rectus Muscle Force (T_{LR}) of the Vertical Bottom Muscle Force*

$$-K_{SE}\left(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}\right)\sin\theta_{LR} + F_{LR}\sin\theta_{LR} + K_{LT}\left(\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}\right)\sin\theta_{LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}\sin\theta_{LR} = 0$$

Taking into consideration that $\theta_{HR\_LR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_SE\_LR}$, the following equations can be derived:

$$\theta_{HR\_LR} - \Delta\theta_{HR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR} - \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$\theta_{HR\_LR} - \Delta\theta_{HR} - \theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR} = \theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$-K_{SE}\left(\theta_{HR\_LR} - \Delta\theta_{HR} - \theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR}\right) + F_{LR} + K_{LT}\left(\theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}\right) - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

$$-K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) + \left(F_{LR} - K_{SE}\left(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}\right) + K_{LT}\theta_{HR\_LT\_LR}\right) - K_{LT}\Delta\theta_{HR\_LT\_LR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

Assigning:

$$\hat{F}_{LR} = F_{LR} - K_{SE}\left(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}\right) + K_{LT}\theta_{HR\_LT\_LR}$$

$$-K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) = -\hat{F}_{LR} + K_{LT}\Delta\theta_{HR\_LT\_LR} + B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

The new equation for T_{LR} can be written as:

$$T_{LR} = -K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right)$$

$$T_{LR} = -\hat{F}_{LR} + K_{LT}\Delta\theta_{HR\_LT\_LR} + B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}} = \Delta\theta_{HR\_LT\_LR}$$

$$T_{LR} = -\hat{F}_{LR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}\right) + B_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{LR} = -\frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{LR} = -K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right)$$

3.4.3. *Medial Rectus Muscle Force (T_{MR}) of the Vertical Bottom Muscle Force*

$$K_{SE}\left(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}\right)\sin\theta_{MR} - F_{MR}\sin\theta_{MR} - K_{LT}\left(\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}\right)\sin\theta_{MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}\sin\theta_{MR} = 0$$

Taking into consideration that $\theta_{HR\_MR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_SE\_MR}$, the following equations can be derived:

$$\theta_{HR\_MR} + \Delta\theta_{HR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR} + \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$\theta_{HR\_MR} + \Delta\theta_{HR} - \theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR} = \theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$K_{SE}\left(\theta_{HR\_MR} + \Delta\theta_{HR} - \theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR}\right) - F_{MR} - K_{LT}\left(\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}\right) - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

$$K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right) - F_{MR} - K_{SE}\theta_{HR\_LT\_MR} + K_{SE}\theta_{HR\_MR} - K_{LT}\theta_{HR\_LT\_MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

Assigning:

$$\hat{F}_{MR} = F_{MR} + K_{SE}\theta_{HR\_LT\_MR} - K_{SE}\theta_{HR\_MR} + K_{LT}\theta_{HR\_LT\_MR}$$

$$K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right) = \hat{F}_{MR} + K_{LT}\Delta\theta_{HR\_LT\_MR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$-K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right) = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

The new equation for T_{MR} can be written as:

$$T_{MR} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right)$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR}$$

$$T_{MR} = -\hat{F}_{MR} - K_{LT}\left(\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR}\right) - B_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{MR} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{MR} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right)$$

3.4.4. Formulating Vertical Bottom Muscle Force ($T_{VR\_B\_MF}$)

$$T_{VR\_B\_MF} = T_{MR} + T_{IR} + T_{LR}$$

$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR}\right) - K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}\right) - K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right)$$

$$T_{VR\_B\_MF} = K_{SE}\Delta\theta_{HR\_LT\_MR} - K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR}\right) - K_{SE}\Delta\theta_{HR\_LT\_LR}$$

$$T_{VR\_B\_MF} = -K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_LR}) \tag{11}$$

$$T_{VR\_B\_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} - \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{VR\_B\_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{VR\_B\_MF} = -\left(\frac{\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{LR}}{K_{SE} + K_{LT}}\right)K_{SE} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \tag{12}$$

3.5. Oculomotor Plant Mechanical Model Equations

As the agonist, the lateral rectus applies horizontal right muscle force (HR_R_MF) to the eye globe according to equations (3) and (4). These equations can be combined as follows:

$$K_{SE}(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}) = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \tag{13}$$

The medial, superior, and inferior recti act collectively as the antagonist, applying horizontal left muscle force (HR_L_MF) to the eye globe according to equations (5) and (6). These equations can be combined as follows:

$$K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_SR}\right) = \frac{\left(\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{SR}\right)}{K_{SE} + K_{LT}}K_{SE} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_MR} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} \tag{14}$$

Applying Newton's second law, the sum of all forces acting on the eye globe horizontally equals the acceleration of the eye globe multiplied by the inertia of the eye globe: $J\Delta\ddot{\theta}_{HR} = T_{HR\_R\_MF} - T_{HR\_L\_MF} - K_P\Delta\theta_{HR} - B_P\Delta\dot{\theta}_{HR}$, where J is the eye globe's inertial mass, $\Delta\theta$ is the rotation of the eye globe, $\Delta\dot{\theta}$ is the velocity of eye rotation, and $\Delta\ddot{\theta}$ is the acceleration of eye rotation. Substituting the equations for $T_{HR\_R\_MF}$ and $T_{HR\_L\_MF}$:

$$J\Delta\ddot{\theta}_{HR} = K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) - K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_SR}\right) - K_p\Delta\theta_{HR} - B_p\Delta\dot{\theta}_{HR} \quad \textbf{(15)}$$

As the agonist, the superior rectus applies vertical top muscle force (VR_T_MF) to the eye globe according to equations (9) and (10). These equations can be combined as follows:

$$K_{SE}\left(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}\right) = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(16)}$$

The inferior, medial, and lateral recti act collectively as the antagonist, applying vertical bottom muscle force (VR_B_MF) to the eye globe according to equations (11) and (12). These equations can be combined as follows:

$$K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_LR}\right) = \left(\frac{\hat{F}_{MR} + \hat{F}_{IR} + \hat{F}_{LR}}{K_{SE} + K_{LT}}\right)K_{SE} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \quad \textbf{(17)}$$

Applying Newton's second law, the sum of all forces acting on the eye globe horizontally equals the acceleration of the eye globe multiplied by the inertia of the eye globe: $J\Delta\ddot{\theta}_{VR} = T_{VR\_T\_MF} - T_{VR\_B\_MF} - K_P\Delta\theta_{VR} - B_P\Delta\dot{\theta}_{VR}$, where J is the eye globe's inertial mass, $\Delta\theta$ is the rotation of the eye globe, $\Delta\dot{\theta}$ is the velocity of eye rotation, and $\Delta\ddot{\theta}$ is the acceleration of eye rotation. Substituting the equations for $T_{VR\_T\_MF}$ and $T_{VR\_B\_MF}$:

$$J\Delta\ddot{\theta}_{VR} = K_{SE}\left(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}\right) - K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_LR}\right) - K_p\Delta\theta_{VR} - B_p\Delta\dot{\theta}_{VR} \quad \textbf{(18)}$$

Two differential equations can be added:

$$\Delta\dot{\theta}_{HR} = \Delta\dot{\theta}_{HR} \quad \textbf{(19)}$$

$$\Delta\dot{\theta}_{VR} = \Delta\dot{\theta}_{VR} \quad \textbf{(20)}$$

The dynamics of the active-state tension for the horizontal agonist muscle at time t can be described with the following equation:

$$\dot{F}_{HR\_AG}(t) = \frac{N_{AG} - \hat{F}_{HR\_AG}(t)}{\tau_{AG\_SAC}} \quad \textbf{(21)}$$

The dynamics of the active-state tension for the horizontal antagonist muscle at time t can be described with the following equation:

$$\dot{F}_{HR\_ANT}(t) = \frac{N_{ANT} - \hat{F}_{HR\_ANT}(t)}{\tau_{ANT\_SAC}} \quad \textbf{(22)}$$

The dynamics of the active-state tension for the vertical agonist muscle at time t can be described with the following equation:

$$\dot{F}_{VR\_AG}(t) = \frac{N_{AG} - \hat{F}_{VR\_AG}(t)}{\tau_{AG\_SAC}} \quad \textbf{(23)}$$

The dynamics of the active-state tension for the vertical antagonist muscle at time t can be described with the following equation:

$$\dot{F}_{VR\_ANT}(t) = \frac{N_{ANT} - \hat{F}_{VR\_ANT}(t)}{\tau_{ANT\_SAC}} \quad \textbf{(24)}$$

Then, the OPMM can be described by the twelve differential equations (13) – (24), with twelve variables: $\Delta\Theta_{HR\_LT\_LR}$, $\Delta\Theta_{HR\_LT\_MR}$, $\Delta\Theta_{VR\_LT\_SR}$, $\Delta\Theta_{VR\_LT\_IR}$, $\Delta\Theta_{HR}$, $\Delta\Theta_{VR}$, $\hat{F}_{MR}$, $\hat{F}_{LR}$, $\hat{F}_{SR}$, $\hat{F}_{IR}$, $\Delta\dot{\theta}_{HR}$, $\Delta\dot{\theta}_{VR}$.

4. Left-downward Movement

For brevity, detailed descriptions of muscle roles/relations have been omitted, as they are symmetric to, and can be inferred from, those provided in the previous section.

4.1. Horizontal Left Muscle Force (HR_L_MF)

We can write the equation of force generated by contraction of the medial rectus:

$$T_{HR\_L\_MF} = -F_{MR}\cos\theta_{MR} - K_{LT}(\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR})\cos\theta_{MR} + B_{AG}\Delta\dot{\theta}_{LT\_MR}\cos\theta_{MR} \quad \textbf{(25)}$$

Resisting the contraction, the series elasticity component propagates the contractile force by pulling the eye globe with the same force:

$$T_{HR\_L\_MF} = -K_{SE}(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR})\cos\theta_{MR} \quad \textbf{(26)}$$

Equations (25) and (26) can be used to calculate the force $T_{HR\_L\_MF}$ in terms of the eye rotation $\Delta\theta_{HR}$ and displacement $\Delta\theta_{HR\_LT\_MR}$ of the length-tension component.

$$-K_{SE}(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR})\,cos\,\theta_{MR} + F_{MR}\,cos\,\theta_{MR} + K_{LT}(\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR})\,cos\,\theta_{MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}\,cos\,\theta_{MR} = 0$$

Taking into consideration that $\theta_{HR\_MR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_SE\_MR}$, the following equations can be derived:

$$\theta_{HR\_MR} - \Delta\theta_{HR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$\theta_{HR\_MR} - \Delta\theta_{HR} - \theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR} = \theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$-K_{SE}(\theta_{HR\_MR} - \Delta\theta_{HR} - \theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}) + F_{MR} + K_{LT}(\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR}) - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

$$-K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}) + (F_{MR} - K_{SE}(\theta_{HR\_MR} - \theta_{HR\_LT\_MR}) + K_{LT}\theta_{HR\_LT\_MR}) - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

Assigning:

$$\hat{F}_{MR} = F_{MR} - K_{SE}(\theta_{HR\_MR} - \theta_{HR\_LT\_MR}) + K_{LT}\theta_{HR\_LT\_MR}$$

$$-K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}) = -\hat{F}_{MR} + K_{LT}\Delta\theta_{HR\_LT\_MR} + B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

The new equation for $T_{HR\_L\_MF}$ can be written as:

$$T_{HR\_L\_MF} = -K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR})$$

$$T_{HR\_L\_MF} = -\hat{F}_{MR} + K_{LT}\Delta\theta_{HR\_LT\_MR} + B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$\Delta\theta_{HR} - \frac{T_{HR\_L\_MF}}{K_{SE}} = \Delta\theta_{HR\_LT\_MR}$$

$$T_{HR\_L\_MF} = -\hat{F}_{MR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{HR\_L\_MF}}{K_{SE}}\right) + B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{HR\_L\_MF} = -K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}) \quad \textbf{(27)}$$

$$T_{HR\_L\_MF} = -\frac{\ddot{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} \quad \textbf{(28)}$$

4.2. Horizontal Right Muscle Force (HR_R_MF)

$$T_{LR} = F_{LR}\cos\theta_{LR} + K_{LT}(\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR})\cos\theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}\cos\theta_{LR}$$

$$T_{IR} = F_{IR}\sin\theta_{IR} + K_{LT}(\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR})\sin\theta_{IR} - B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}\sin\theta_{IR}$$

$$T_{SR} = F_{SR}\sin\theta_{SR} + K_{LT}(\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_SR})\sin\theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_LR}\sin\theta_{SR}$$

$$T_{LR} = K_{SE}(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR})\cos\theta_{LR}$$

$$T_{IR} = K_{SE}(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR})\sin\theta_{IR}$$

$$T_{SR} = K_{SE}(\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR})\sin\theta_{SR}$$

These equations can be used to calculate the forces T_{LR}, T_{IR}, and T_{SR} in terms of the eye rotation, $\Delta\theta_{HR}$ and $\Delta\theta_{VR}$, and displacement of the length-tension component of each muscle, $\Delta\theta_{HR\_LT\_LR}$, $\Delta\theta_{VR\_LT\_IR}$, and $\Delta\theta_{VR\_LT\_SR}$.

4.2.1. Lateral Rectus Muscle Force (T_{LR}) of the Horizontal Right Muscle Force

$$-K_{SE}(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR})\cos\theta_{LR} + F_{LR}\cos\theta_{LR} + K_{LT}(\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR})\cos\theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}\cos\theta_{LR} = 0$$

Taking into consideration that $\theta_{HR\_LR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_SE\_LR}$, the following equations can be derived:

$$\theta_{HR\_LR} + \Delta\theta_{HR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR} + \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$\theta_{HR\_LR} + \Delta\theta_{HR} - \theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR} = \theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$-K_{SE}(\theta_{HR\_LR} + \Delta\theta_{HR} - \theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}) + F_{LR} + K_{LT}(\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR}) + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

$$-K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}) + F_{LR} - K_{SE}(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}) + K_{LT}\theta_{HR\_LT\_LR} + K_{LT}\Delta\theta_{HR\_LT\_LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

Assigning:

$$\hat{F}_{LR} = F_{LR} - K_{SE}(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}) + K_{LT}\theta_{HR\_LT\_LR}$$

$$K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}) = \hat{F}_{LR} + K_{LT}\Delta\theta_{HR\_LT\_LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}$$

The new equation for T_{LR} can be written as:

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR})$$

$$\Delta\theta_{HR\_LT\_LR} = \Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}$$

$$T_{LR} = \hat{F}_{LR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}\right) + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{LR} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR})$$

4.2.2. Superior Rectus Muscle Force (T_{SR}) of the Horizontal Right Muscle Force

$$-K_{SE}(\theta_{VR\_SE\_SR}+\Delta\theta_{VR\_SE\_SR})\sin\theta_{SR}+F_{SR}\sin\theta_{SR}+K_{LT}(\theta_{VR\_LT\_SR}+\Delta\theta_{VR\_LT\_SR})\sin\theta_{SR}+B_{ANT}\Delta\dot{\theta}_{VR\_LT\_LR}\sin\theta_{SR}=0$$

Taking into consideration that $\theta_{VR\_SR} = \theta_{VR\_LT\_SR} + \theta_{VR\_SE\_SR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_SE\_SR}$, the following equations can be derived:

$$\theta_{VR\_SR}+\Delta\theta_{VR}=\theta_{VR\_LT\_SR}+\theta_{VR\_SE\_SR}+\Delta\theta_{VR\_LT\_SR}+\Delta\theta_{VR\_SE\_SR}$$

$$\theta_{VR\_SR}+\Delta\theta_{VR}-\theta_{VR\_LT\_SR}-\Delta\theta_{VR\_LT\_SR}=\theta_{VR\_SE\_SR}+\Delta\theta_{VR\_SE\_SR}$$

$$-K_{SE}(\theta_{VR\_SR}+\Delta\theta_{VR}-\theta_{VR\_LT\_SR}-\Delta\theta_{VR\_LT\_SR})+F_{SR}+K_{LT}(\theta_{VR\_LT\_SR}+\Delta\theta_{VR\_LT\_SR})+B_{ANT}\Delta\dot{\theta}_{VR\_LT\_LR}=0$$

$$-K_{SE}(\Delta\theta_{VR}-\Delta\theta_{VR\_LT\_SR})+F_{SR}-K_{SE}(\theta_{VR\_SR}-\theta_{VR\_LT\_SR})+K_{LT}\theta_{VR\_LT\_SR}+K_{LT}\Delta\theta_{VR\_LT\_SR}+B_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}=0$$

Assigning:

$$\hat{F}_{SR}=F_{SR}-K_{SE}(\theta_{VR\_SR}-\theta_{VR\_LT\_SR})+K_{LT}\theta_{VR\_LT\_SR}$$

$$K_{SE}(\Delta\theta_{VR}-\Delta\theta_{VR\_LT\_SR})=\hat{F}_{SR}+K_{LT}\Delta\theta_{VR\_LT\_SR}+B_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

The new equation for T_{SR} can be written as:

$$T_{SR}=K_{SE}(\Delta\theta_{VR}-\Delta\theta_{VR\_LT\_SR})$$

$$\Delta\theta_{VR\_LT\_SR}=\Delta\theta_{VR}-\frac{T_{SR}}{K_{SE}}$$

$$T_{SR}=S_{SR}+K_{LT}\left(\Delta\theta_{VR}-\frac{T_{SR}}{K_{SE}}\right)+B_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{SR}=\frac{\hat{F}_{SR}K_{SE}}{K_{SE}+K_{LT}}+\frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE}+K_{LT}}+\hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{SR}=K_{SE}(\Delta\theta_{VR}-\Delta\theta_{VR\_LT\_SR})$$

4.2.3. Inferior Rectus Muscle Force (T_{IR}) of the Horizontal Right Muscle Force

$$-K_{SE}(\theta_{VR\_SE\_IR}+\Delta\theta_{VR\_SE\_IR})\sin\theta_{IR}+F_{IR}\sin\theta_{IR}+K_{LT}(\theta_{VR\_LT\_IR}-\Delta\theta_{VR\_LT\_IR})\sin\theta_{IR}-B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}\sin\theta_{IR}=0$$

Taking into consideration that $\theta_{VR\_IR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_SE\_IR}$, the following equations can be derived:

$$\theta_{VR\_IR}-\Delta\theta_{VR}=\theta_{VR\_LT\_IR}+\theta_{VR\_SE\_IR}-\Delta\theta_{VR\_LT\_IR}+\Delta\theta_{VR\_SE\_IR}$$

$$\theta_{VR\_IR}-\Delta\theta_{VR}-\theta_{VR\_LT\_IR}+\Delta\theta_{VR\_LT\_IR}=\theta_{VR\_SE\_IR}+\Delta\theta_{VR\_SE\_IR}$$

$$-K_{SE}(\theta_{VR\_IR}-\Delta\theta_{VR}-\theta_{VR\_LT\_IR}+\Delta\theta_{VR\_LT\_IR})+F_{IR}+K_{LT}(\theta_{VR\_LT\_IR}-\Delta\theta_{VR\_LT\_IR})-B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}=0$$

$$-K_{SE}(\Delta\theta_{VR\_LT\_IR}-\Delta\theta_{VR})+F_{IR}-K_{SE}(\theta_{VR\_IR}-\theta_{VR\_LT\_IR})+K_{LT}\theta_{VR\_LT\_IR}-K_{LT}\Delta\theta_{VR\_LT\_IR}-B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}=0$$

Assigning:

$$\hat{F}_{IR}=F_{IR}-K_{SE}(\theta_{VR\_IR}-\theta_{VR\_LT\_IR})+K_{LT}\theta_{VR\_LT\_IR}$$

$$K_{SE}(\Delta\theta_{VR\_LT\_IR}-\Delta\theta_{VR})=\hat{F}_{IR}-K_{LT}\Delta\theta_{VR\_LT\_IR}-B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}$$

The new equation for T_{IR} can be written as:

$$T_{IR}=K_{SE}(\Delta\theta_{VR\_LT\_IR}-\Delta\theta_{VR})$$

$$\frac{T_{IR}}{K_{SE}}+\Delta\theta_{VR}=\Delta\theta_{VR\_LT\_IR}$$

$$T_{IR} = \hat{F}_{IR} - K_{LT}\left(\frac{T_{IR}}{K_{SE}} + \Delta\theta_{VR}\right) - B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{IR} = \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{IR} = K_{SE}(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR})$$

4.2.4. *Formulating Horizontal Right Muscle Force ($T_{HR\_R\_MF}$)*

$$T_{HR\_R\_MF} = T_{LR} + T_{IR} + T_{SR}$$

$$T_{HR\_R\_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}) + K_{SE}(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}) + K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR})$$

$$T_{HR\_R\_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_SR})$$

$$T_{HR\_R\_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} + \frac{\hat{F}_{IR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} + \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{HR\_R\_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{IR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(29)}$$

$$T_{HR\_R\_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_SR}) \quad \textbf{(30)}$$

4.3. *Vertical Bottom Muscle Force ($T_{VR\_B\_MF}$)*

We can write the equation of force generated by contraction of the inferior rectus:

$$T_{VR\_B\_MF} = -F_{IR}\cos\theta_{IR} - K_{LT}(\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR})\cos\theta_{IR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}\cos\theta_{IR} \quad \textbf{(31)}$$

Resisting the contraction, the series elasticity component propagates the contractile force by pulling the eye globe with the same force:

$$T_{VR\_B\_MF} = -K_{SE}(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR})\cos\theta_{IR} \quad \textbf{(32)}$$

Equations (31) and (32) can be used to calculate the force $T_{VR\_B\_MF}$ in terms of the eye rotation $\Delta\theta_{VR}$ and displacement $\Delta\theta_{VR\_LT\_IR}$ of the length-tension component.

$$K_{SE}(\theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR})\cos\theta_{IR} - F_{IR}\cos\theta_{IR} - K_{LT}(\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR})\cos\theta_{IR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}\cos\theta_{IR} = 0$$

Taking into consideration that $\theta_{VR\_IR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_SE\_IR}$, the following equations can be derived:

$$\theta_{VR\_IR} - \Delta\theta_{VR} = \theta_{VR\_LT\_IR} + \theta_{VR\_SE\_IR} - \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_SE\_IR}$$

$$\theta_{VR\_IR} - \Delta\theta_{VR} - \theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR} = \theta_{VR\_SE\_IR} + \Delta\theta_{VR\_SE\_IR}$$

$$K_{SE}(\theta_{VR\_IR} - \Delta\theta_{VR} - \theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_IR}) - F_{IR} - K_{LT}(\theta_{VR\_LT\_IR} - \Delta\theta_{VR\_LT\_IR}) + B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} = 0$$

$$K_{SE}(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}) - F_{IR} + K_{SE}(\theta_{VR\_IR} - \theta_{VR\_LT\_IR}) - K_{LT}\theta_{VR\_LT\_IR} + K_{LT}\Delta\theta_{VR\_LT\_IR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} = 0$$

Assigning:

$$\hat{F}_{IR} = F_{IR} + K_{SE}(\theta_{VR\_LT\_IR} - \theta_{VR\_IR}) + K_{LT}\theta_{VR\_LT\_IR}$$

$$-K_{SE}(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}) = -\hat{F}_{IR} + K_{LT}\Delta\theta_{VR\_LT\_IR} + B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}$$

The new equation for $T_{VR\_B\_MF}$ can be written as:

$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}\right)$$

$$\Delta\theta_{VR} - \frac{T_{VR\_B\_MF}}{K_{SE}} = \Delta\theta_{VR\_LT\_IR}$$

$$T_{VR\_B\_MF} = -\hat{F}_{IR} + K_{LT}\left(\Delta\theta_{VR} - \frac{T_{VR\_B\_MF}}{K_{SE}}\right) + B_{AG}\Delta\dot{\theta}_{VR\_LT\_IR}$$

$$T_{VR\_B\_MF} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE}+K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} \quad \textbf{(33)}$$

$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}\right) \quad \textbf{(34)}$$

4.4. Vertical Top Muscle Force ($T_{VR\_T\_MF}$)

$$T_{LR} = F_{LR}\sin\theta_{LR} + K_{LT}\left(\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR}\right)\sin\theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}\sin\theta_{LR}$$

$$T_{MR} = F_{MR}\sin\theta_{MR} + K_{LT}\left(\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR}\right)\sin\theta_{MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}\sin\theta_{MR}$$

$$T_{SR} = F_{SR}\cos\theta_{SR} + K_{LT}\left(\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_SR}\right)\cos\theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_LR}\cos\theta_{SR}$$

$$T_{LR} = K_{SE}\left(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}\right)\sin\theta_{LR}$$

$$T_{MR} = K_{SE}\left(\theta_{MR\_SE\_MR} + \Delta\theta_{MR\_SE\_MR}\right)\sin\theta_{MR}$$

$$T_{SR} = K_{SE}\left(\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR}\right)\cos\theta_{SR}$$

These equations can be used to calculate the forces T_{LR}, T_{MR}, and T_{SR} in terms of the eye rotation, $\Delta\theta_{HR}$ and $\Delta\theta_{VR}$, and displacement of the length-tension component of each muscle, $\Delta\theta_{HR\_LT\_LR}$, $\Delta\theta_{HR\_LT\_MR}$, and $\Delta\theta_{VR\_LT\_SR}$.

4.4.1. Lateral Rectus Muscle Force (T_{LR}) of the Vertical Top Muscle Force

$$-K_{SE}\left(\theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}\right)\sin\theta_{LR} + F_{LR}\sin\theta_{LR} + K_{LT}\left(\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR}\right)\sin\theta_{LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}\sin\theta_{LR} = 0$$

Taking into consideration that $\theta_{HR\_LR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_SE\_LR}$, the following equations can be derived:

$$\theta_{HR\_LR} + \Delta\theta_{HR} = \theta_{HR\_LT\_LR} + \theta_{HR\_SE\_LR} + \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$\theta_{HR\_LR} + \Delta\theta_{HR} - \theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR} = \theta_{HR\_SE\_LR} + \Delta\theta_{HR\_SE\_LR}$$

$$-K_{SE}\left(\theta_{HR\_LR} + \Delta\theta_{HR} - \theta_{HR\_LT\_LR} - \Delta\theta_{HR\_LT\_LR}\right) + F_{LR} + K_{LT}\left(\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_LR}\right) + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

$$-K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}\right) + F_{LR} - K_{SE}\left(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}\right) + K_{LT}\theta_{HR\_LT\_LR} + K_{LT}\Delta\theta_{HR\_LT\_LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} = 0$$

Assigning:

$$\hat{F}_{LR} = F_{LR} - K_{SE}\left(\theta_{HR\_LR} - \theta_{HR\_LT\_LR}\right) + K_{LT}\theta_{HR\_LT\_LR}$$

$$K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}\right) = \hat{F}_{LR} + K_{LT}\Delta\theta_{HR\_LT\_LR} + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}$$

The new equation for T_{LR} can be written as:

$$T_{LR} = K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}\right)$$

$$\Delta\theta_{HR\_LT\_LR} = \Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}$$

$$T_{LR} = \hat{F}_{LR} + K_{LT}\left(\Delta\theta_{HR} - \frac{T_{LR}}{K_{SE}}\right) + B_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{LR} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR}$$

$$T_{LR} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR})$$

4.4.2. Superior Rectus Muscle Force (T_{SR}) of the Vertical Top Muscle Force

$$-K_{SE}(\theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR})\cos\theta_{SR} + F_{SR}\cos\theta_{SR} + K_{LT}(\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_SR})\cos\theta_{SR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_LR}\cos\theta_{SR} = 0$$

Taking into consideration that $\theta_{VR\_SR} = \theta_{VR\_LT\_SR} + \theta_{VR\_SE\_SR}$ and $\Delta\theta_{VR} = \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_SE\_SR}$, the following equations can be derived:

$$\theta_{VR\_SR} + \Delta\theta_{VR} = \theta_{VR\_LT\_SR} + \theta_{VR\_SE\_SR} + \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_SE\_SR}$$

$$\theta_{VR\_SR} + \Delta\theta_{VR} - \theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR} = \theta_{VR\_SE\_SR} + \Delta\theta_{VR\_SE\_SR}$$

$$-K_{SE}(\theta_{VR\_SR} + \Delta\theta_{VR} - \theta_{VR\_LT\_SR} - \Delta\theta_{VR\_LT\_SR}) + F_{SR} + K_{LT}(\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_SR}) + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_LR} = 0$$

$$-K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR}) + F_{SR} - K_{SE}(\theta_{VR\_SR} - \theta_{VR\_LT\_SR}) + K_{LT}\theta_{VR\_LT\_SR} + K_{LT}\Delta\theta_{VR\_LT\_SR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} = 0$$

Assigning:

$$\hat{F}_{SR} = F_{SR} - K_{SE}(\theta_{VR\_SR} - \theta_{VR\_LT\_SR}) + K_{LT}\theta_{VR\_LT\_SR}$$

$$K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR}) = \hat{F}_{SR} + K_{LT}\Delta\theta_{VR\_LT\_SR} + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

The new equation for T_{SR} can be written as:

$$T_{SR} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR})$$

$$\Delta\theta_{VR\_LT\_SR} = \Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}$$

$$T_{SR} = \hat{F}_{SR} + K_{LT}\left(\Delta\theta_{VR} - \frac{T_{SR}}{K_{SE}}\right) + B_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{SR} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{SR} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR})$$

4.4.3. Medial Rectus Muscle Force (T_{MR}) of the Vertical Top Muscle Force

$$-K_{SE}(\theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR})\sin\theta_{MR} + F_{MR}\sin\theta_{MR} + K_{LT}(\theta_{HR\_LT\_MR} - \Delta\theta_{VR\_LT\_MR})\sin\theta_{MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}\sin\theta_{MR} = 0$$

Taking into consideration that $\theta_{HR\_MR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR}$ and $\Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_SE\_MR}$, the following equations can be derived:

$$\theta_{HR\_MR} - \Delta\theta_{HR} = \theta_{HR\_LT\_MR} + \theta_{HR\_SE\_MR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$\theta_{HR\_MR} - \Delta\theta_{HR} - \theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR} = \theta_{HR\_SE\_MR} + \Delta\theta_{HR\_SE\_MR}$$

$$-K_{SE}(\theta_{HR\_MR} - \Delta\theta_{HR} - \theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_MR}) + F_{MR} + K_{LT}(\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_MR}) - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

$$-K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}) + (F_{MR} - K_{SE}(\theta_{HR\_MR} - \theta_{HR\_LT\_MR}) + K_{LT}\theta_{HR\_LT\_MR}) - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} = 0$$

Assigning:

$$\hat{F}_{MR} = F_{MR} - K_{SE}(\theta_{HR\_MR} - \theta_{HR\_LT\_MR}) + K_{LT}\theta_{HR\_LT\_MR}$$

$$K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}) = \hat{F}_{MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

The new equation for T_{MR} can be written as:

$$T_{MR} = K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR})$$

$$T_{MR} = \hat{F}_{MR} - K_{LT}\Delta\theta_{HR\_LT\_MR} - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR} = \Delta\theta_{HR\_LT\_MR}$$

$$T_{MR} = \hat{F}_{MR} - K_{LT}\left(\frac{T_{MR}}{K_{SE}} + \Delta\theta_{HR}\right) - B_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{MR} = \frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR}$$

$$T_{MR} = K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR})$$

4.4.4. Formulating Vertical Top Muscle Force ($T_{VR\_T\_MF}$)

$$T_{VR\_T\_MF} = T_{LR} + T_{MR} + T_{SR}$$

$$T_{VR\_T\_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} + \frac{\hat{F}_{MR}K_{SE}}{K_{SE} + K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} + \frac{\hat{F}_{SR}K_{SE}}{K_{SE} + K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR}$$

$$T_{VR\_T\_MF} = K_{SE}(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR}) + K_{SE}(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}) + K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR})$$

$$T_{VR\_T\_MF} = \frac{(\hat{F}_{LR} + \hat{F}_{MR} + \hat{F}_{SR})}{K_{SE} + K_{LT}}K_{SE} + \frac{\Delta\theta_{VR}K_{SE}K_{LT}}{K_{SE} + K_{LT}} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(35)}$$

$$T_{VR\_T\_MF} = K_{SE}(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR\_LT\_LR}) \quad \textbf{(36)}$$

5. Muscle Model Equations

The equations for the remaining directions of eye globe rotation can be inferred from the equations and muscle model diagrams presented in the previous sections, and follow the general form:

$$T_{Plane\_Direction\_MF} = \pm K_{SE}(\Delta\theta_{Plane\_LT\_AG} - \Delta\theta_{Plane}) \quad \textbf{(37)}$$

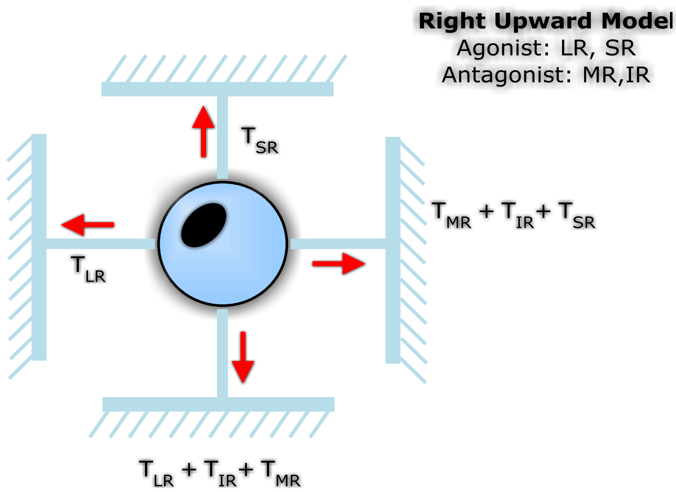
$$T_{Plane\_Direction\_MF} = \pm\frac{\hat{F}_{AG}K_{SE}}{K_{SE} + K_{LT}} \mp \frac{\Delta\theta_{Plane}K_{SE}K_{LT}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{Plane\_LT\_AG} \quad \textbf{(38)}$$

$$T_{Plane\_Direction\_MF} = \pm K_{SE}(\Delta\theta_{Plane} - \Delta\theta_{Plane\_LT\_ANT} - \Delta\theta_{PerpendicularPlane\_LT\_ANT} + \Delta\theta_{PerpendicularPlane\_LT\_AG}) \quad \textbf{(39)}$$

$$\begin{aligned} T_{Plane\_Direction\_MF} = & \pm\frac{K_{SE}(\hat{F}_{ANT} + \hat{F}_{PerpendicularAG} + \hat{F}_{PerpendicularANT})}{K_{SE} + K_{LT}} \pm \frac{\Delta\theta_{Plane}K_{SE}K_{LT}}{K_{SE} + K_{LT}} \mp \hat{B}_{AG}\Delta\dot{\theta}_{PerpendicularPlane\_LT\_AG} \\ & \pm \hat{B}_{ANT}\Delta\dot{\theta}_{Plane\_LT\_ANT} \pm \hat{B}_{ANT}\Delta\dot{\theta}_{PerpendicularPlane\_LT\_ANT} \end{aligned} \quad \textbf{(40)}$$

By modifying the plane (horizontal or vertical), force direction (top, bottom, left, or right), and agonist/antagonist muscle pairs (lateral, medial, superior, or inferior recti) associated with these equations, it is possible to describe the dynamics of eye globe rotation within the two-dimensional plane.

5.1. Right-upward Movement



	Horizontal (HR)	**Vertical (VR)**
Force Direction	Right (R)	Top (T)
Agonist Muscle (AG)	Lateral Rectus (LR)	Superior Rectus (SR)
Antagonist Muscle (ANT)	Medial Rectus (MR)	Inferior Rectus (IR)

Applying equations (37) – (40) to the horizontal and vertical planes, we obtain:

$$T_{HR\_R\_MF} = K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) \quad \textbf{(41)}$$

$$T_{HR\_R\_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \quad \textbf{(42)}$$

$$T_{HR\_L\_MF} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{VR\_LT\_IR} + \Delta\theta_{VR\_LT\_SR}\right) \quad \textbf{(43)}$$

$$T_{HR\_L\_MF} = -\frac{\left(\hat{F}_{SR}+\hat{F}_{MR}+\hat{F}_{IR}\right)}{K_{SE}+K_{LT}}K_{SE} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} \quad \textbf{(44)}$$

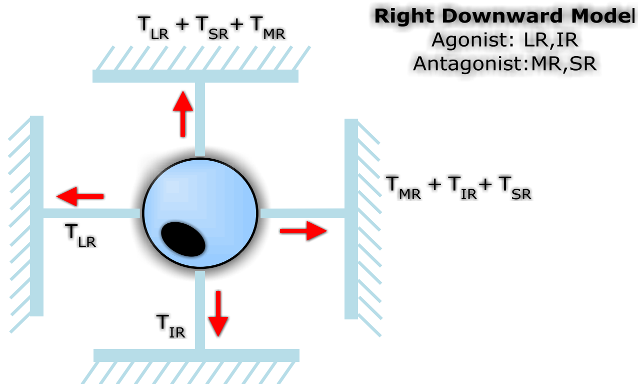
$$T_{VR\_T\_MF} = K_{SE}\left(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}\right) \quad \textbf{(45)}$$

$$T_{VR\_T\_MF} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(46)}$$

$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_LR}\right) \quad \textbf{(47)}$$

$$T_{VR\_B\_MF} = -\frac{\left(\hat{F}_{LR}+\hat{F}_{IR}+\hat{F}_{MR}\right)}{K_{SE}+K_{LT}}K_{SE} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} \quad \textbf{(48)}$$

5.2. Right-downward Movement



	Horizontal (HR)	**Vertical (VR)**
Force Direction	Right (R)	Bottom (B)
Agonist Muscle (AG)	Lateral Rectus (LR)	Inferior Rectus (IR)
Antagonist Muscle (ANT)	Medial Rectus (MR)	Superior Rectus (SR)

Applying equations (37) – (40) to the horizontal and vertical planes, we obtain:

$$T_{HR\_R\_MF} = K_{SE}\left(\Delta\theta_{HR\_LT\_LR} - \Delta\theta_{HR}\right) \quad \textbf{(49)}$$

$$T_{HR\_R\_MF} = \frac{\hat{F}_{LR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} \quad \textbf{(50)}$$

$$T_{HR\_L\_MF} = -K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_MR} - \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_IR}\right) \quad \textbf{(51)}$$

$$T_{HR\_L\_MF} = -\frac{\left(\hat{F}_{IR}+\hat{F}_{MR}+\hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} - \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(52)}$$

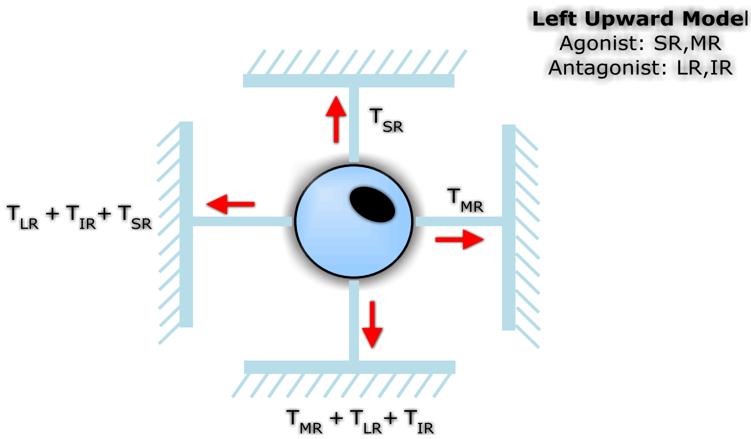
$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}\right) \quad \textbf{(53)}$$

$$T_{VR\_B\_MF} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE}+K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} \quad \textbf{(54)}$$

$$T_{VR\_T\_MF} = K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR} - \Delta\theta_{HR\_LT\_MR} + \Delta\theta_{HR\_LT\_LR}\right) \quad \textbf{(55)}$$

$$T_{VR\_T\_MF} = \frac{\left(\hat{F}_{LR}+\hat{F}_{MR}+\hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_LR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_MR} \quad \textbf{(56)}$$

5.3. Left-upward Movement



	Horizontal (HR)	Vertical (VR)
Force Direction	Left (L)	Top (T)
Agonist Muscle (AG)	Medial Rectus (MR)	Superior Rectus (SR)
Antagonist Muscle (ANT)	Lateral Rectus (LR)	Inferior Rectus (IR)

Applying equations (37) – (40) to the horizontal and vertical planes, we obtain:

$$T_{HR\_L\_MF} = -K_{SE}\left(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}\right) \quad \textbf{(57)}$$

$$T_{HR\_L\_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE}+K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} \quad \textbf{(58)}$$

$$T_{HR\_R\_MF} = K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR} - \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_IR}\right) \quad \textbf{(59)}$$

$$T_{HR\_R\_MF} = \frac{\left(\hat{F}_{IR} + \hat{F}_{LR} + \hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(60)}$$

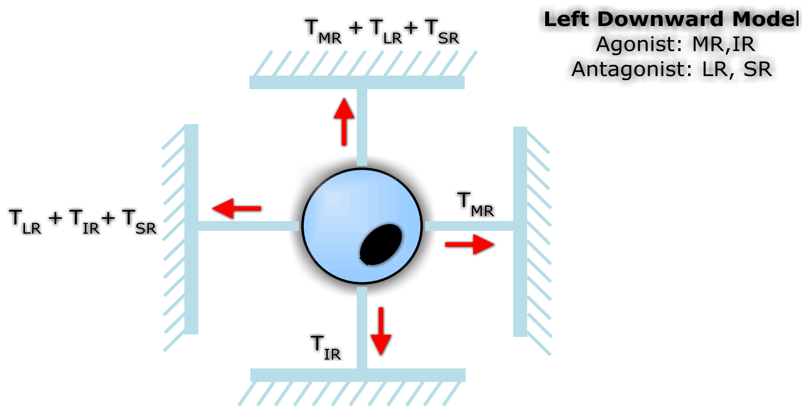
$$T_{VR\_T\_MF} = K_{SE}\left(\Delta\theta_{VR\_LT\_SR} - \Delta\theta_{VR}\right) \quad \textbf{(61)}$$

$$T_{VR\_T\_MF} = \frac{\hat{F}_{SR}K_{SE}}{K_{SE}+K_{LT}} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(62)}$$

$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_IR} - \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_MR}\right) \quad \textbf{(63)}$$

$$T_{VR\_B\_MF} = -\frac{\left(\hat{F}_{MR} + \hat{F}_{LR} + \hat{F}_{IR}\right)}{K_{SE}+K_{LT}}K_{SE} - \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} - \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_IR} - \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} \quad \textbf{(64)}$$

5.4. Left-downward Movement



	Horizontal (HR)	**Vertical (VR)**
Force Direction	Left (L)	Bottom (B)
Agonist Muscle (AG)	Medial Rectus (MR)	Inferior Rectus (IR)
Antagonist Muscle (ANT)	Lateral Rectus (LR)	Superior Rectus (SR)

Applying equations (37) – (40) to the horizontal and vertical planes, we obtain:

$$T_{HR\_L\_MF} = -K_{SE}\left(\Delta\theta_{HR\_LT\_MR} - \Delta\theta_{HR}\right) \quad \textbf{(65)}$$

$$T_{HR\_L\_MF} = -\frac{\hat{F}_{MR}K_{SE}}{K_{SE}+K_{LT}} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} \quad \textbf{(66)}$$

$$T_{HR\_R\_MF} = K_{SE}\left(\Delta\theta_{HR} - \Delta\theta_{HR\_LT\_LR} - \Delta\theta_{VR\_LT\_SR} + \Delta\theta_{VR\_LT\_IR}\right) \quad \textbf{(67)}$$

$$T_{HR\_R\_MF} = \frac{\left(\hat{F}_{IR}+\hat{F}_{LR}+\hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} + \frac{\Delta\theta_{HR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} \quad \textbf{(68)}$$

$$T_{VR\_B\_MF} = -K_{SE}\left(\Delta\theta_{VR\_LT\_IR} - \Delta\theta_{VR}\right) \quad \textbf{(69)}$$

$$T_{VR\_B\_MF} = -\frac{\hat{F}_{IR}K_{SE}}{K_{SE}+K_{LT}} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} + \hat{B}_{AG}\Delta\dot{\theta}_{VR\_LT\_IR} \quad \textbf{(70)}$$

$$T_{VR\_T\_MF} = K_{SE}\left(\Delta\theta_{VR} - \Delta\theta_{VR\_LT\_SR} - \Delta\theta_{HR\_LT\_LR} + \Delta\theta_{HR\_LT\_MR}\right) \quad \textbf{(71)}$$

$$T_{VR\_T\_MF} = \frac{\left(\hat{F}_{MR}+\hat{F}_{LR}+\hat{F}_{SR}\right)}{K_{SE}+K_{LT}}K_{SE} + \frac{\Delta\theta_{VR}K_{SE}}{K_{SE}+K_{LT}} - \hat{B}_{AG}\Delta\dot{\theta}_{HR\_LT\_MR} + \hat{B}_{ANT}\Delta\dot{\theta}_{VR\_LT\_SR} + \hat{B}_{ANT}\Delta\dot{\theta}_{HR\_LT\_LR} \quad \textbf{(72)}$$