# Hash Tables

Chapter 20

CS 3358
Summer II 2013

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Sections 20.1, 20.2, 20.3, 20.4 (not 20.4.2), 20.5



## What are hash tables?

- A Hash Table is used to implement a **set**, providing basic operations in constant time:
  - insert
  - remove (optional)
  - find
  - makeEmpty (need not be constant time)
- The table uses a function that maps an object in the set to its location in the table.
- The function is called a **hash function**.



## Using a hash function

| | values |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | Empty |
| [ 3 ] | 7803 |
| [ 4 ] | Empty |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**HandyParts company makes no more than 100 different parts. But the parts all have four digit numbers.**

**This hash function can be used to store and retrieve parts in an array.**

**Hash(partNum) = partNum % 100**



## Placing elements in the array

| | values |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | Empty |
| [ 3 ] | 7803 |
| [ 4 ] | Empty |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**Use the hash function**

**Hash(partNum) = partNum % 100**

**to place the element with part number 5502 in the array.**

# Placing elements in the array

values

| | |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | 5502 |
| [ 3 ] | |
| [ 4 ] | 7803 |
| | Empty |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**Next place part number 6702 in the array.**

**Hash(partNum) = partNum % 100**

**6702 % 100 = 2**

**But values[2] is already occupied.**

**COLLISION OCCURS**



# How to resolve the collision?

values

| | |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | 5502 |
| [ 3 ] | |
| [ 4 ] | 7803 |
| | Empty |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**One way is by linear probing. This uses the following function**

**(HashValue + 1) % 100**

**repeatedly until an empty location is found for part number 6702.**



# Resolving the collision

values

| | |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | 5502 |
| [ 3 ] | |
| [ 4 ] | 7803 |
| | Empty |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**Still looking for a place for 6702 using the function**

**(HashValue + 1) % 100**



# Collision resolved

values

| | |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | 5502 |
| [ 3 ] | |
| [ 4 ] | 7803 |
| | Empty |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**Part 6702 can be placed at the location with index 4.**

# Collision resolved

| | values |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | 5502 |
| [ 3 ] | 7803 |
| [ 4 ] | 6702 |
| . | . |
| . | . |
| . | . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**Part 6702 is placed at the location with index 4.**

**Where would the part with number 4598 be placed using linear probing?**



# Hashing concepts

- **Hash Table**: where objects are stored by according to their key (usually an array)
  - **key**: attribute of an object used for searching/ sorting
  - number of valid keys usually greater than number of slots in the table
  - number of keys in use usually much smaller than table size.
- **Hash function**: maps keys to a Table index
- **Collision**: when two separate keys hash to the same location



# Hashing concepts

- **Collision resolution**: method for finding an open spot in the table for a key that has collided with another key already in the table.
- **Load Factor**: the fraction of the hash table that is full
  - may be given as a percentage: 50%
  - may be given as a fraction in the range from 0 to 1, as in: .5



# Hash Function

- Goals:
  - computation should be fast
  - should minimize collisions (good distribution)
- Some issues:
  - should depend on ALL of the key (not just the last 2 digits or first 3 characters, which may not themselves be well distributed)

# Hash Function

- Final step of hash function is usually:

  temp % size
  - temp is some intermediate result
  - size is the hash table size
  - ensures the value is a valid location in the table
- Picking a value for size:
  - Bad choices:
    - a power of 2: then the result is only the lowest order bits of temp (not based on whole key)
    - a power of 10: result is only lowest order digits of decimal number
  - Good choices: prime numbers



# Hash Function: string keys

- If the key is not a number, hash function must transform it to a number, to mod by the size
- Method 1: Add up ascii values

```
int hash (string key, int tableSize) {
  int hashVal = 0;
  for (int i=0; i<key.length(); i++)
    hashVal = hashVal + key[i];
  return hashVal % tableSize;
}
```

- different permutations of same chars have same hash value ("cat" and "act" have same value)
- large tableSize and short key length do not distribute well:

> if tableSize is 10007 and keys are 8 characters long:
> since ascii values are <=127, hash produces values between 0 and 127*8 = 1016
> all falling in first 1/10th of the table



# Hash Function: string keys

- Method 2: Multiply each char by a power of 128

$$hash = k[0]*128^3 + k[1]*128^2 + k[2]*128^1 + k[3]*128^0$$

This equivalent to (which avoids large intermediate results):

$$hash = (((k[0]*128 + k[1])*128 + k[2])*128 + k[3])*128$$

```
int hash (string key, int tableSize) {
  int hashVal = 0;
  for (int i=0; i<key.length(); i++)
    hashVal = (hashVal * 128 + key[i]) % tableSize;
  return hashVal;
}
```

- now "cat" and "act" map to different values (most likely)
- but now we get really big numbers (overflow)
- we take mod of intermediate results to reduce overflow
- but mod is expensive . . .



# Hash Function: string keys

- could just allow overflow, take mod at the end
- but repeated multiplying by 128 tends to shift early chars out to the left (potentially leading to more collisions)

- Method 3: Multiply each char by a power of 37

$$hash = (((k[0]*37 + k[1])*37 + k[2])*37 + k[3])*37$$

```
int hash (string key, int tableSize) {
  int hashVal = 0;
  for (int i=0; i<key.length(); i++)
    hashVal = hashVal * 37 + key[i];
  return hashVal % tableSize;
}
```

- compromise.  37 is prime.  Has good distribution.
- "au" and "bP" map to the same value, but collisions are less common than in method 1.

# Collision Resolution: 1. Linear Probing

- Insert: When there is a collision, search sequentially for the next available slot
- Find: if the key is not at the hashed location, keep searching sequentially for it.
  - if it reaches an empty slot, the key is not found
- Problem: if the the table is somewhat full, it may take a long time to find the open slot.
  - May not be O(1) any more
- Problem: Removing an element in the middle of a chain



# Linear Probing: Example

- Insert: 89, 18, 49, 58, 69, hash(k) = k mod 10

Probing function (attempt i): $h_i(K)$ = (hash(K) + **i**) % tablesize

| | Empty Table | After 89 | After 18 | After 49 | After 58 | After 69 |
|---|---|---|---|---|---|---|
| 0 | | | | 49 | 49 | 49 |
| 1 | | | | | 58 | 58 |
| 2 | | | | | | 69 |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | 18 | 18 | 18 | 18 |
| 9 | | 89 | 89 | 89 | 89 | 89 |

49 is in 0 because 9 was full

58 is in 1 because 8, 9, 0 were full

69 is in 1 because 9, 0 were full



# Linear Probing: delete problem

values

| | values |
|---|---|
| [ 0 ] | Empty |
| [ 1 ] | 4501 |
| [ 2 ] | 5502 |
| [ 3 ] | 7803 |
| [ 4 ] | 6702 |
| . . . | . . . |
| [ 97] | Empty |
| [ 98] | 2298 |
| [ 99] | 3699 |

**Part 6702 was placed at the location with index 4, after colliding with 5502**

**Now remove 7803.**

**Now find 6702 (hash(6702)=2):**
**not at values[2]**
**values[3] is empty, so not found**



# Linear Probing: Lazy deletion

- Don’t remove the deleted object, just mark as deleted
- During find, marked deletions don’t stop the searching
- During insert, the spot may be reused
- If there are a lot of deletions, searching may still take a long time in a “sparse” table

# Linear Probing:
## Primary Clustering

- Cluster: a large, sequential block of occupied slots in the table
- Any key that hashes into the cluster requires excessive attempts to resolve the collision
- If it's during an insert operation, the cluster gets bigger.
- If two clusters are separated by one slot, a single insertion will drastically degrade the future performance
- Primary clustering is a problem at high load factors (90%), not at 50% or less.



# Collision Resolution:
## 2. Quadratic Probing

- An attempt to eliminate primary clustering
- If the hash function returns H, and H is occupied, try H+1, then H+4, then H+9, ...
  - for each attempt i, try $H+i^2$ next.

Probing function (attempt i): $h_i(K) = (hash(K) + i^2)$ % tablesize

- Is it guaranteed to find an empty slot if there is one (like linear probing)?
  - Yes IF: the table size is prime and the load is <= 50%



# Quadratic Probing:
## Example

- Insert: 89, 18, 49, 58, 69, hash(k) = k mod 10

Probing function (attempt i): $h_i(K) = (hash(K) + \mathbf{i^2})$ % tablesize

| | Empty Table | After 89 | After 18 | After 49 | After 58 | After 69 |
|---|---|---|---|---|---|---|
| 0 | | | | 49 | 49 | 49 |
| 1 | | | | | | |
| 2 | | | | | 58 | 58 |
| 3 | | | | | | 69 |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | 18 | 18 | 18 | 18 |
| 9 | | 89 | 89 | 89 | 89 | 89 |

49 is in 0 because 9 was full

58 is in 2 because 8, 8+1=9 were full, (8+4)%10=2 wasn't

69 is in 3 because 9, (9+1)%10=0 were full, (9+4)%10=3 wasn't

Note: smaller clusters



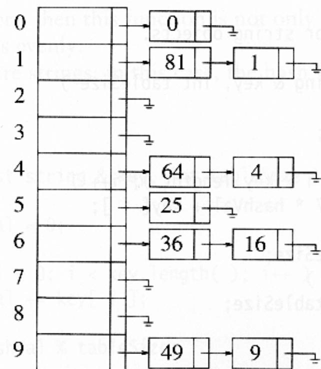
# Quadratic probing:
## expansion of table

- Since the table should be less than 50% full:
- Can the table be expanded if the load factor gets more than 50%?
- Yes.
  - Find the next prime number greater than 2*tableSize, resize to that.
  - Don't just copy all the elements (new tablesize => new hash function)
  - Scan old table for non-empties, and use insert function to add them to new table.
- This is called **rehashing.**

# Collision Resolution:
# 3. Separate chaining

- Use an array of linked lists for the hash table
- Each linked list contains all objects that hashed to that location
  - no collisions

Hash function is still:
h(K) = k % 10






# Separate Chaining

- To insert a an object:
  - compute hash(k)
  - insert at front of list at that location (if empty, make first node)
- To find an object:
  - compute hash(k)
  - search the linked list there for the key of the object
- To delete an object:
  - compute hash(k)
  - search the linked list there for the key of the object
  - if found, remove it



# Separate Chaining

- The load can be 1 or more
  - more than 1 node at each location, still O(1) inserts and finds
  - smaller loads do not improve performance
  - moderately larger loads do not hurt performance
- Disadvantages
  - Memory allocation could be expensive
  - too many nodes at one position can slow operations
- Advantages:
  - deletion is easy
  - don’t have to resize/rehash