Intro to Graphs

CS 5301
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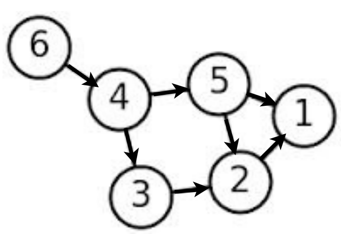
Main + Savitch: 15.1-3

Graph:
non-recursive definition

- **Graph**: set of vertices and edges that connect the vertices.
  - If the edge pairs are ordered, it is called a **directed graph**.
- A vertex w is **adjacent** to vertex v if there is an edge from v to w.
- Edges can have an additional value: a **weight**.
- A **path** is a sequence of vertices connected by edges.
- vertices are also called **nodes**.

Graph:
example



- Vertices: 1, 2, 3, 4, 5, 6
- Edges: (6,4), (4,5) (4,3) (5,2) (3,2) (5,1) (2,1)
- Example of a path: 6,4,5,2

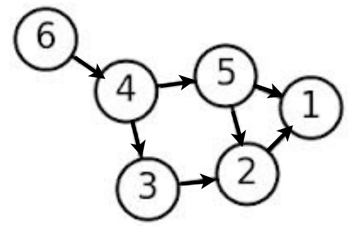
Graph traversal

- Graph traversal: operation that converts the nodes in a graph into a list
  - may encounter a node more than once
- Depth first traversal
  - Print the data from the first node
  - Go to an unvisited adjacent node of the previous node, print its data
  - Repeat until a node has no unvisited adjacent nodes
  - backtrack to a previously visited node, repeat on an unvisited adjacent node, until you backtrack past root.
  - A stack (or recursion) is useful for backtracking

Graph traversal

- Breadth first traversal
  - Begin at the root node,
  - Explore all the adjacent nodes.
  - Then for each of those nearest nodes, explore their adjacent nodes,
  - and so on, until it has visited each node.
  - A queue is useful in keeping track of the unprocessed nodes.

Graph:
example traversals



- DFT: 6 4 5 1 2 3
- BFT: 6 4 3 5 2 1
- More than one correct DFT and BFT

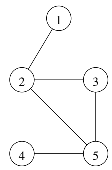
Recursive Depth First Traversal

- Recursive Depth first traversal, given a node
  - Print the data from the node
  - Mark the node as visited
  - For each unvisited adjacent node of the given node, perform Depth first traversal on that node.
  - What is the base case? when there are no unvisited adjacent nodes

Graphs: Representations

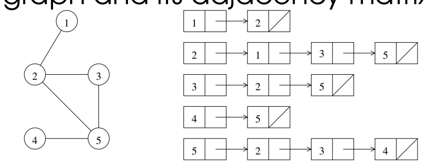
- Adjacency matrix:
  - a two dimensional matrix, rows and columns are nodes.
  - Initialize to 0's.
  - For each edge (v,w) in the graph, set a[v][w] to 1.
    - or set a[v][w] to the weight of the edge from v to w
- Adjacency list:
  - nodes are stored as objects, and each node stores a list of adjacent nodes.
    - and the weight of the edge to the adjacent node
  - uses less space.

Graphs: Representations



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

An undirected graph and its adjacency matrix representation.



An undirected graph and its adjacency list representation.

Graphs: algorithms

- Shortest Path
  - Find the shortest path from one node to another.
  - Paths could be weighted
  - Real world application: Google maps directions
- Traveling salesman problem:
  - Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?
  - Basically exponential run time.